

# Reduction and Identification for Models of Locomotion: an Emerging Systems Theory for Neuromechanics

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# Autonomous machines will pervade our world

Össur



Raytheon



U-Pitt



BDI



Willow



Google



Intuitive

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Gaits & Grasps

Cyber-Physical Systems



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# Dynamic Gaits & Dexterous Grasps

## Dynamic Locomotion

- speeds measured in bodylengths/sec
- scales ranging from cm to m



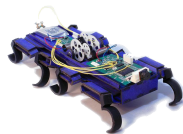
Boston Dynamics, Inc.

## Dexterous Manipulation

- high-precision pick-and-place, repetitive assembly
- fold towels; wash dishes



Koditschek *et al.*



Fearing *et al.*



Willow



Rethink

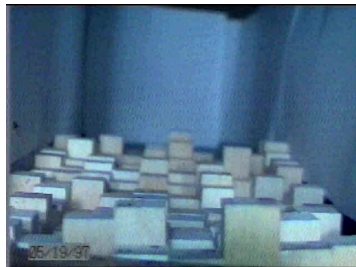
## Future direction

- co-robots in factory & home

# Neuro-Mechanical Systems

## Fundamentals of sensorimotor control

- mechanosensory feedback
- passive self-stabilization



Sponberg & Full

## Design of Assistive Devices

- prosthesis, exoskeleton
- Brain-Machine Interface



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## Future direction

- personalized healthcare



# Cyber-Physical Systems (CPS)

## Automated Healthcare

- teleoperated surgery
- remote diagnosis



Intuitive



Brewer *et al.*

## Human-in-the-Loop

- (semi-)autonomous vehicles
- energy demand response
- social cyber-physical systems



Google



PG&E

## Future direction

- co-design *cyber-and-physical* systems

# Fundamental engineering challenges

## Neuromechanics

### Future direction:

- automated & personalized healthcare

## Gaits & Grasps

### Future direction:

- co-robots in factory & home

## Cyber-Physical Systems

### Future direction:

- co-design *cyber* & *physical*

# Fundamental engineering challenges

## Neuromechanics

### Future direction:

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### Challenges:

- sensitive to environment
- relies on careful calibration

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### Future direction:

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### Challenges:

- distributed, multi-agent
- large scale, multi-physical

# Fundamental engineering challenges

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- automated & personalized healthcare

### Challenges:

- generalization across task/environment
- translation across scale, material, & morphology

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## Common engineering challenge

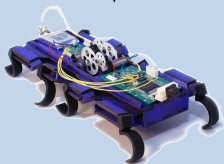
Dynamic interaction between computational & mechanical components

# Dynamic interaction between computation & mechanics

## Gaits & Grasps

### Controller

sensors, computers



### Objects & Obstacles

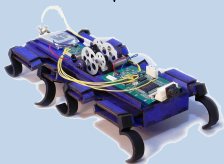
terrain, agents

# Dynamic interaction between computation & mechanics

## Gaits & Grasps

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### Objects & Obstacles

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## Neuromechanics

### Nervous system

sensorimotor elements



### Environment

predator/prey/partner

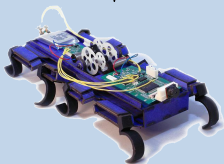


# Dynamic interaction between computation & mechanics

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## Cyber-Physical Sys

### Social planner

distributed sensor web



### Society

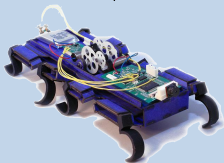
infrastructure, economy

# Dynamic interaction between computation & mechanics

## Gaits & Grasps

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sensors, computers



### Objects & Obstacles

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sensorimotor elements



### Environment

predator/prey/partner

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### Society

infrastructure, economy

Interaction dynamics are piecewise-defined, distributed

Need new framework for modeling and control

# Analytical, computational, & experimental framework

## Neuromechanics

nervous system ↔ environment

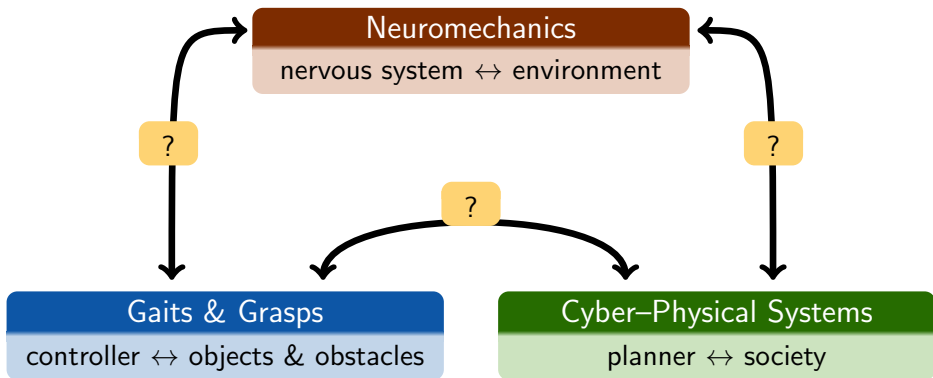
## Gaits & Grasps

controller ↔ objects & obstacles

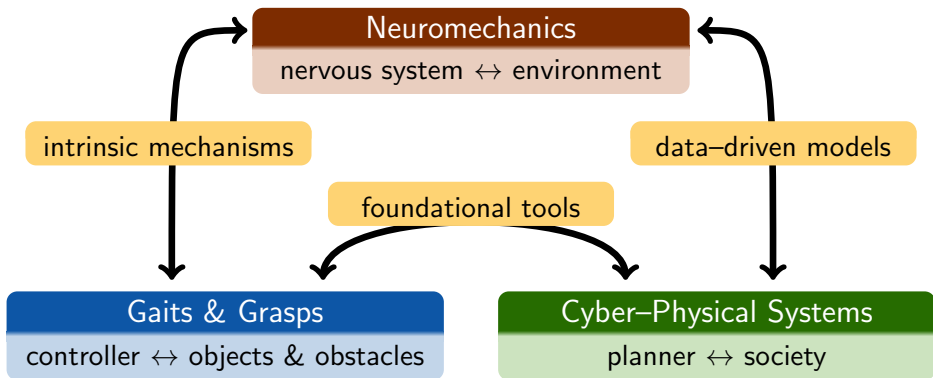
## Cyber-Physical Systems

planner ↔ society

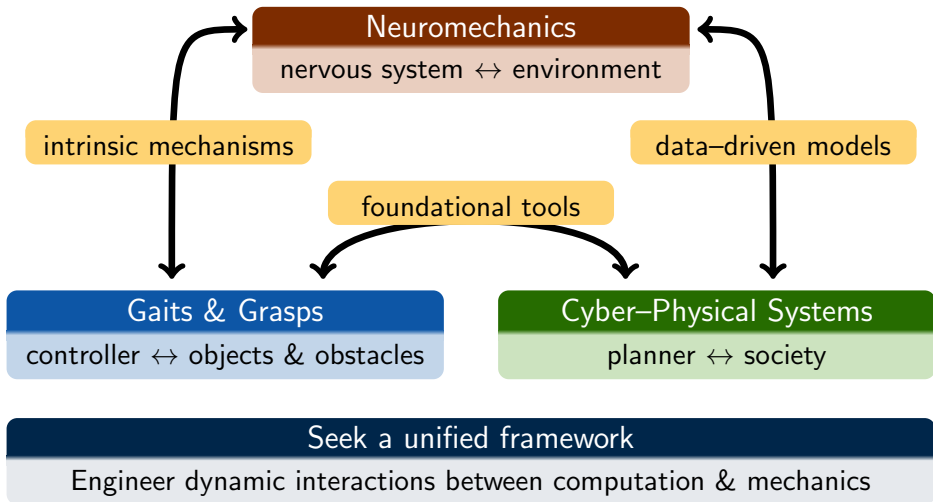
# Analytical, computational, & experimental framework



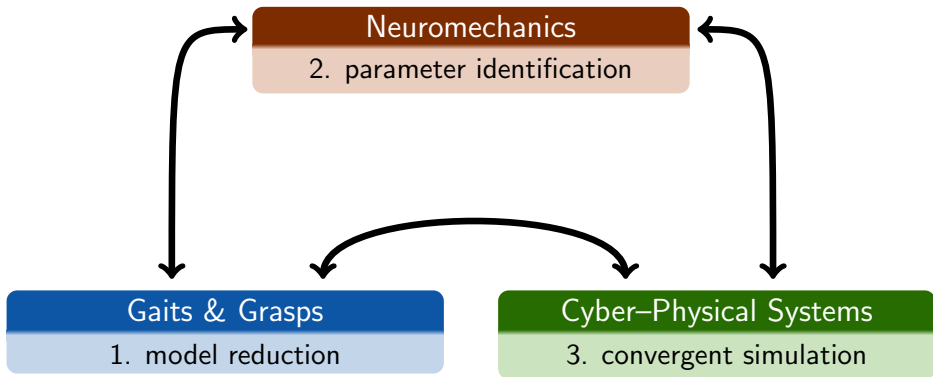
# Analytical, computational, & experimental framework



# Analytical, computational, & experimental framework



# Today's theme: framework for studying locomotion



# Overview of today's talk

## Locomotion

animals are adept at dynamic locomotion

### 1. Reduction

models for periodic gaits generically reduce dimensionality

### 2. Identification

reduction enables scalable algorithm for parameter estimation

### 3. Simulation

convergent numerical simulation for piecewise-defined dynamics

## Future Directions

robust gaits, maneuver synthesis, and inverse modeling



# Animals are extremely adept at dynamic locomotion

## flat-terrain gait



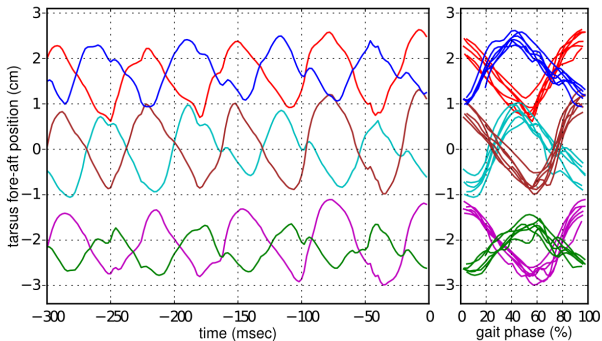
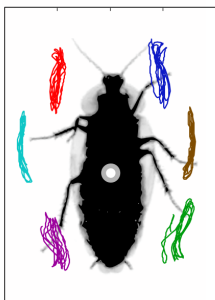
Sandbot RHex robot; Li *et al.* PNAS 2009

## optimized gait



zebra-tailed lizard; Li *et al.* JEB 2012

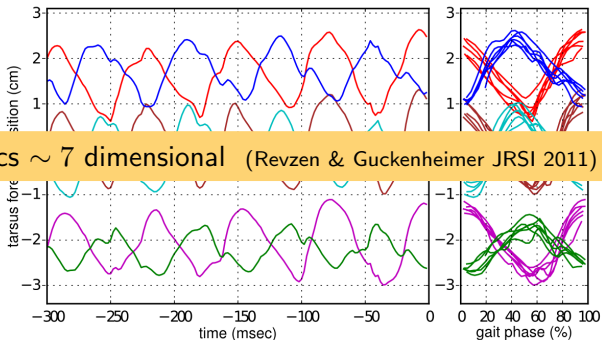
# Empirically, animals use few degrees-of-freedom



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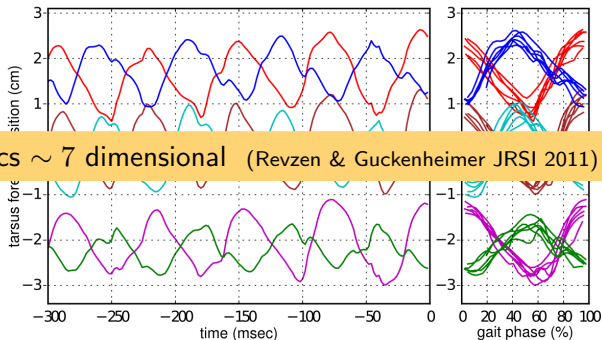
Cockroach dynamics  $\sim 7$  dimensional (Revzen & Guckenheimer JRSI 2011)



# Empirically, animals use few degrees-of-freedom



Cockroach dynamics  $\sim 7$  dimensional (Revzen & Guckenheimer JRSI 2011)



## Mechanisms for reduction in neural or environmental models

Neural synchronization

Cohen et al. J. Math. Bio 1982

Physiological symmetry

Golubitsky et al. Nature 1999

Muscle activation synergy

Ting & Macpherson J. Neurosci. 2005

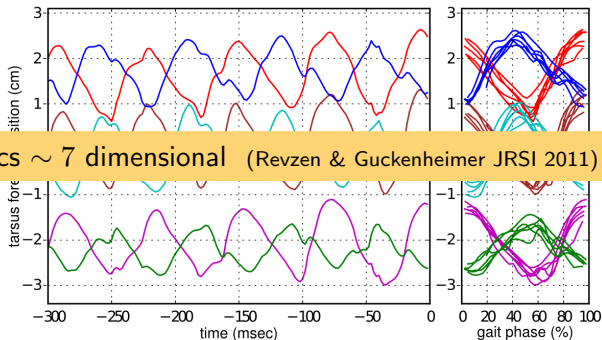
Granular media solidification

Li et al. Science 2013

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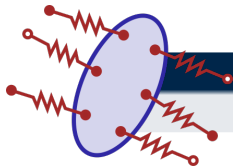
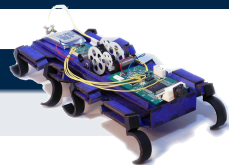
Li et al. Science 2013

Need reduction tool for interaction between body and environment

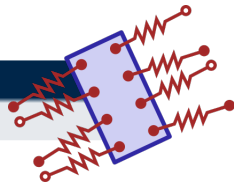
# Use simple models to study animal and robot gaits



physical system  
animal, robot

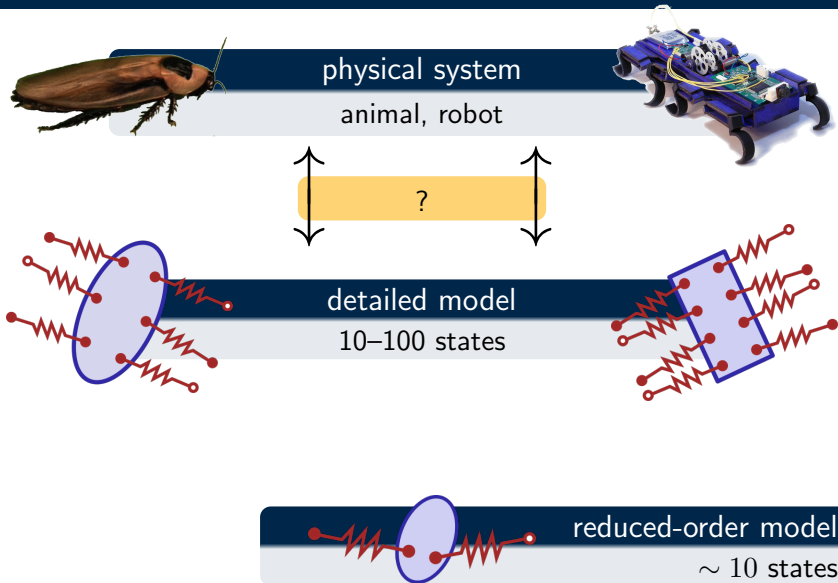


detailed model  
10–100 states

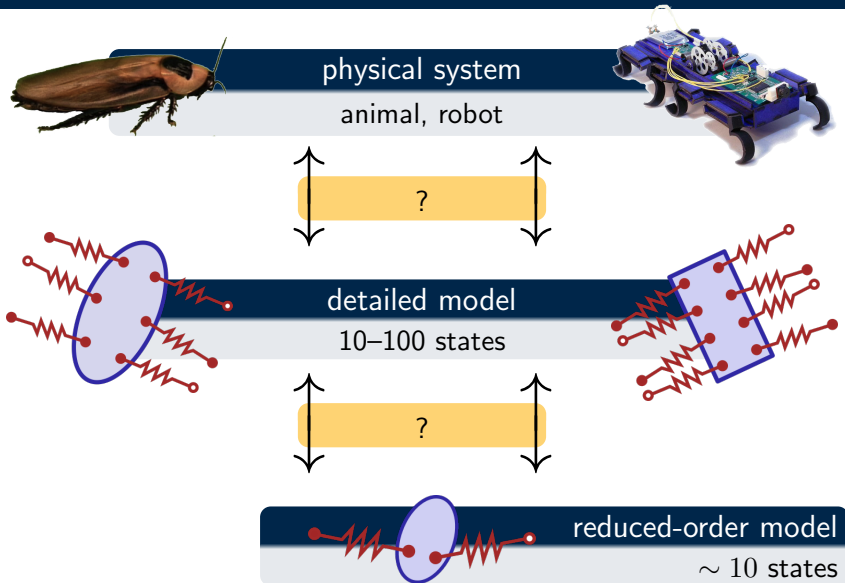


reduced-order model  
~ 10 states

# Use simple models to study animal and robot gaits

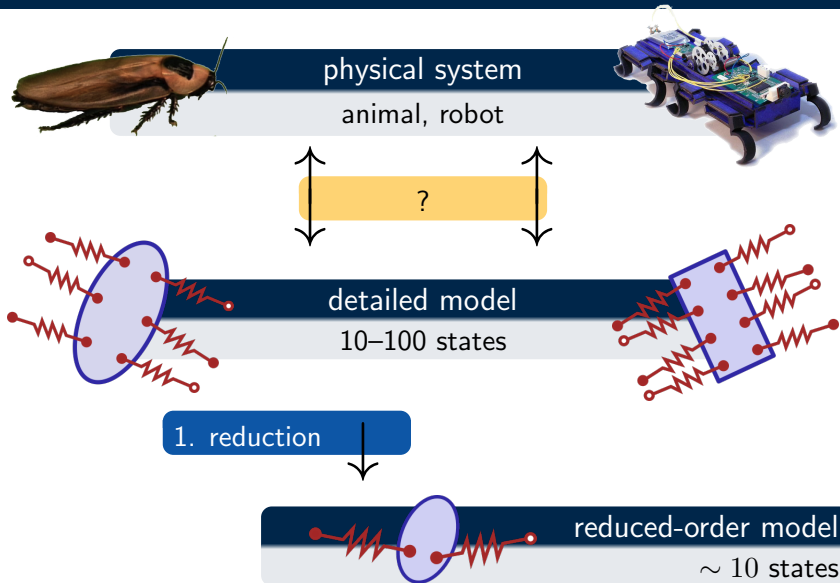


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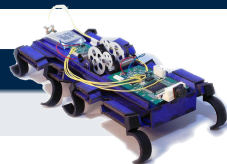
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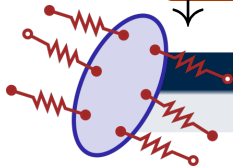
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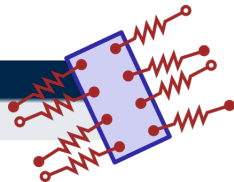
physical system  
animal, robot



2. identification



detailed model  
10–100 states



1. reduction



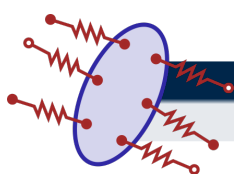
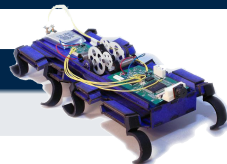
reduced-order model  
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# Model Reduction



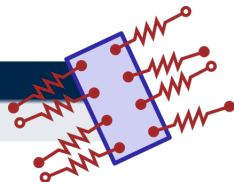
physical system

animal, robot



detailed model

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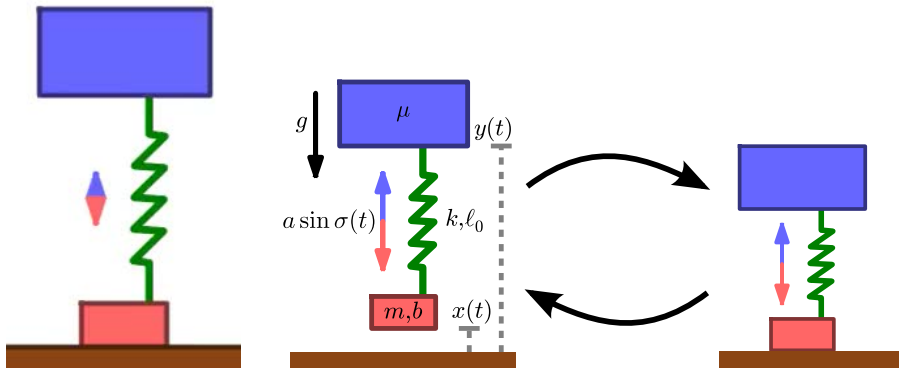
1. reduction



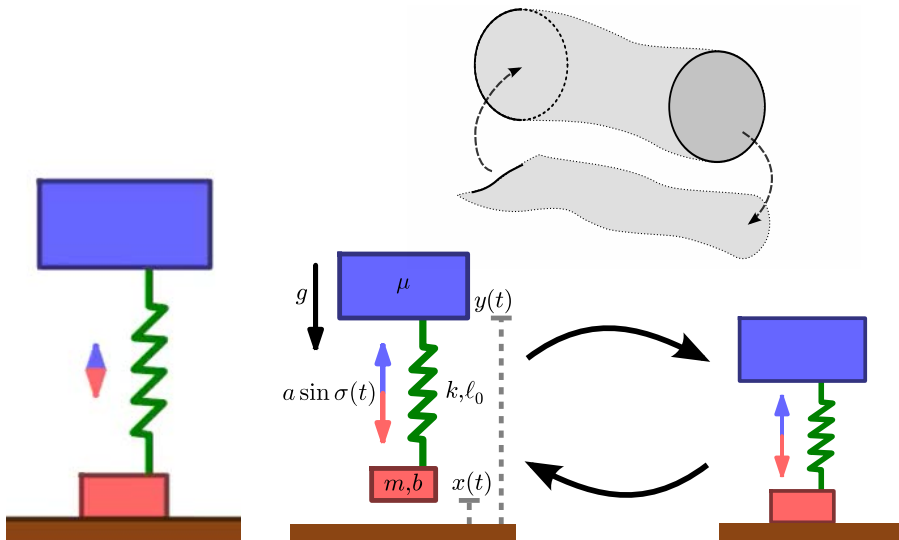
reduced-order model

< 10 states

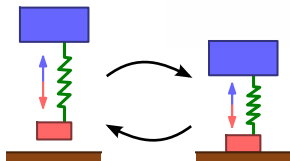
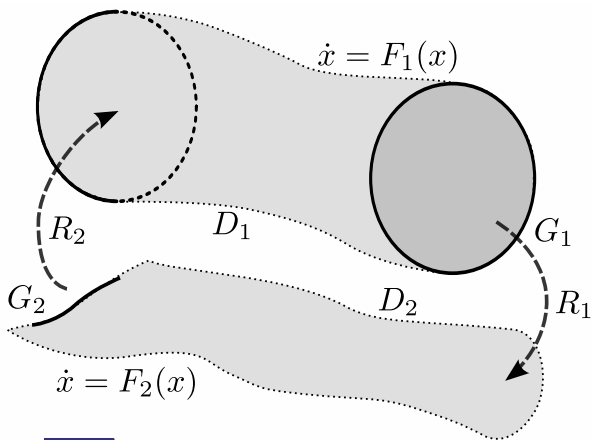
# Dimension loss in vertical hopper



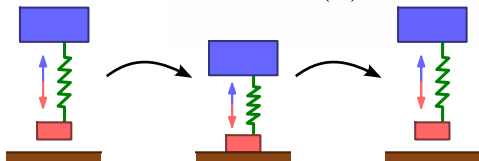
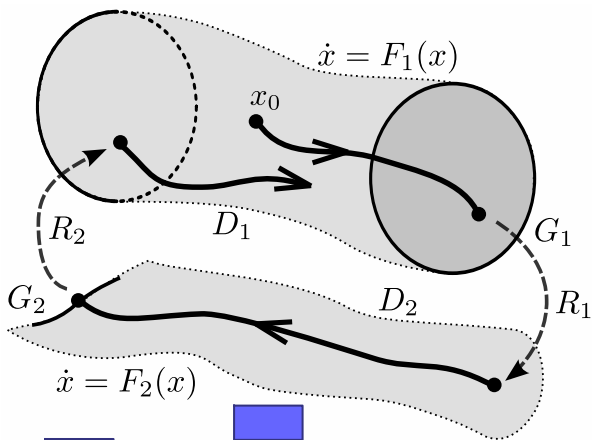
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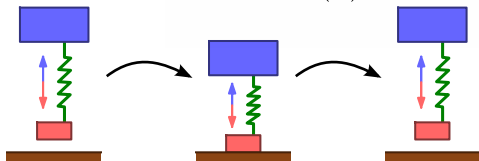
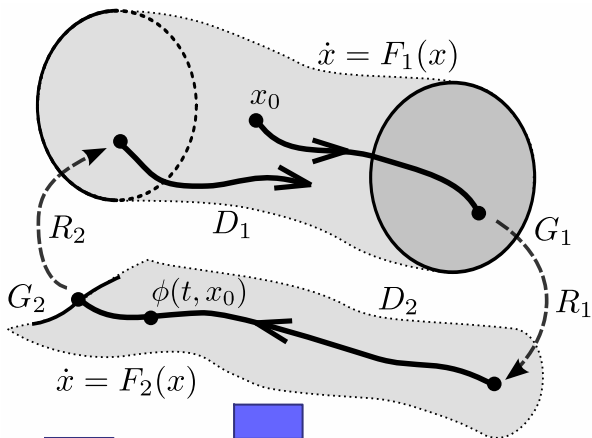
# Hybrid dynamical system



# Trajectory for a hybrid dynamical system

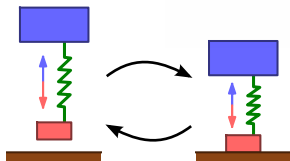
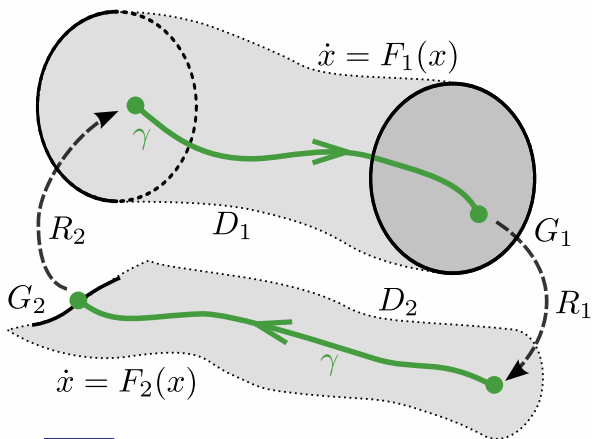


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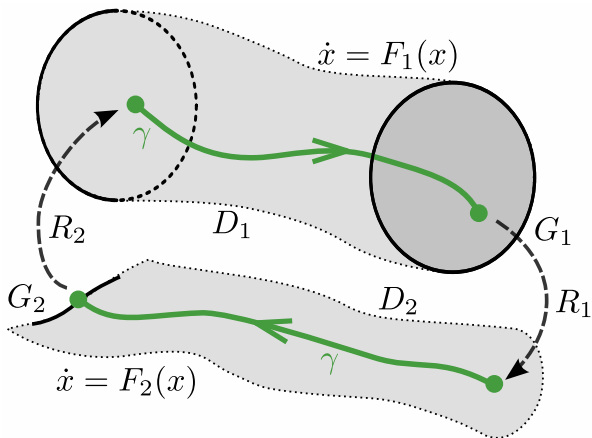




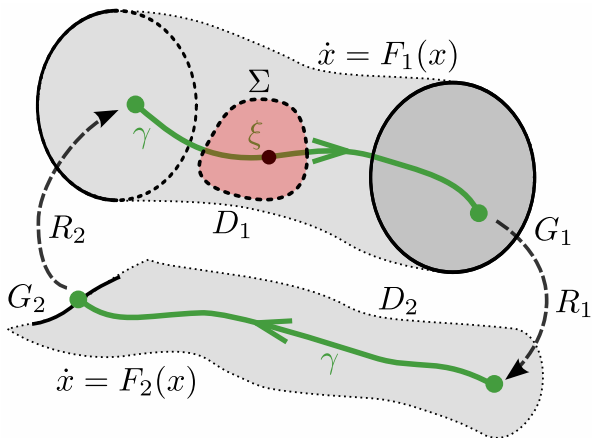
# Periodic orbit $\gamma$ for a hybrid dynamical system



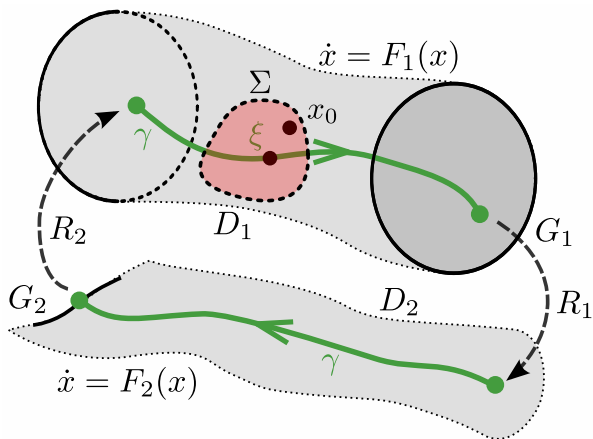
# Poincaré map for periodic orbit $\gamma$



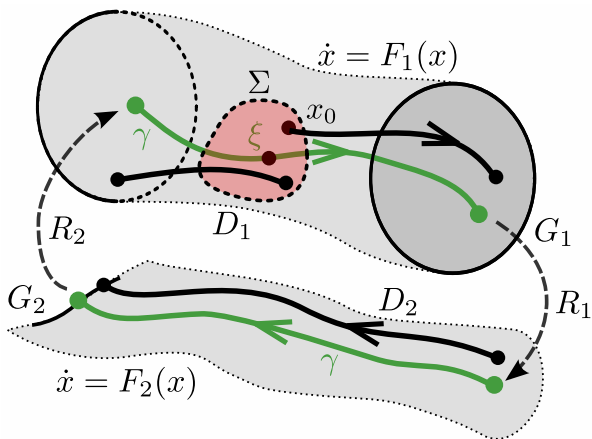
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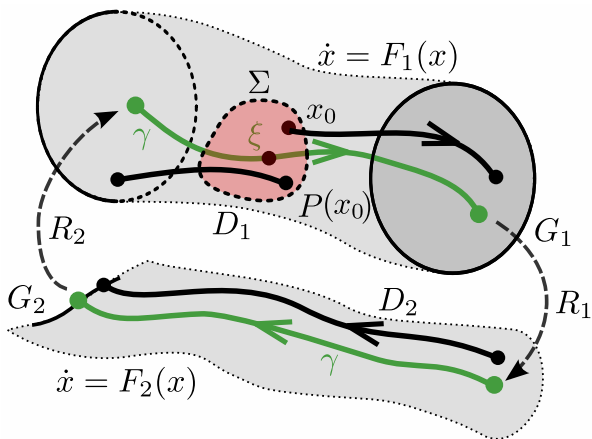
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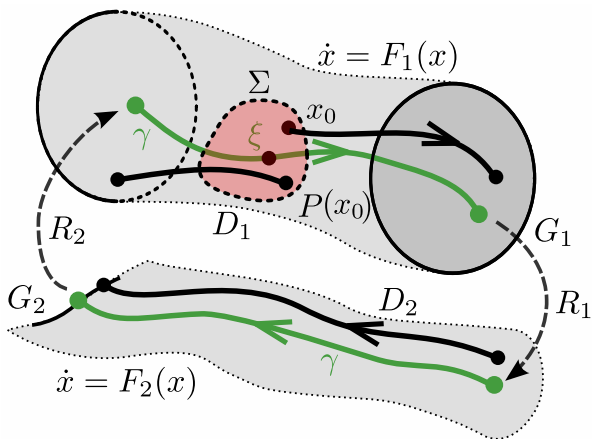
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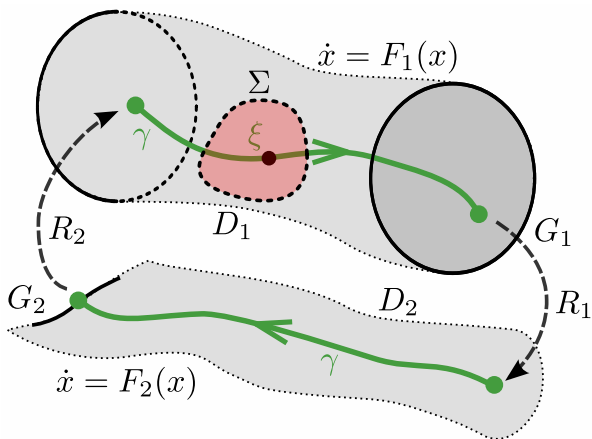
# Poincaré map for periodic orbit $\gamma$



**Theorem (Aizerman & Gantmacher JMAM 1958)**

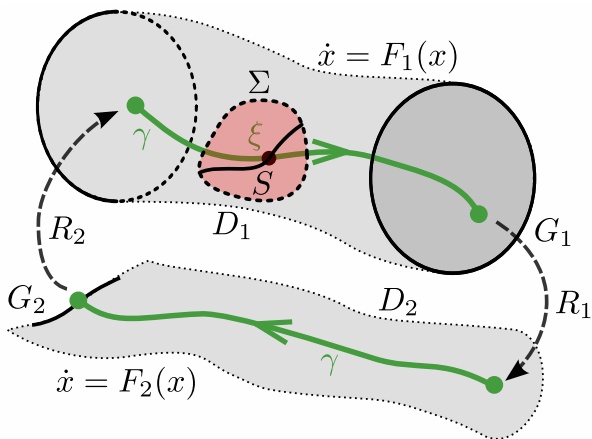
*The Poincaré map  $P$  is smooth in a neighborhood of  $\xi$ .*

# Model reduction near hybrid periodic orbit $\gamma$

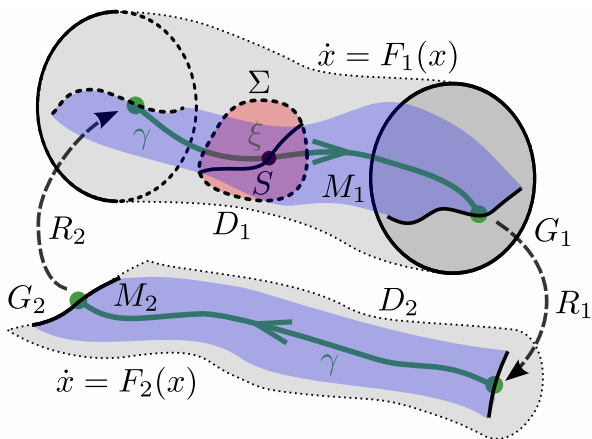




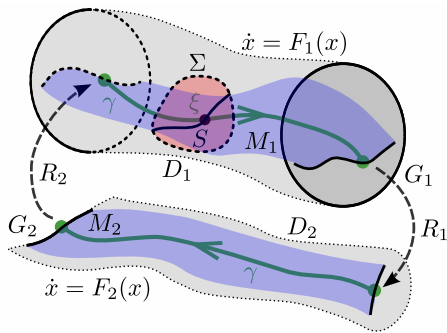
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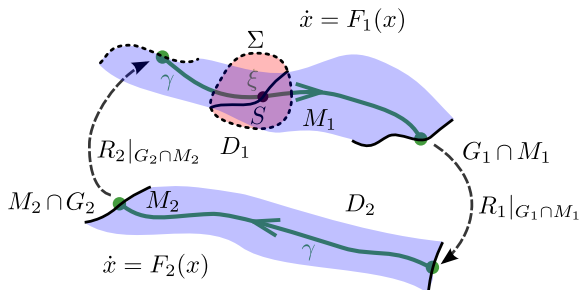


## Theorem (Burden, Revzen, Sastry (arXiv:1308.4158))

Let  $n = \min_j \dim D_j - 1$ . If  $\text{rank } DP^n = r$  near  $\xi$ , then trajectories starting near  $\gamma$  contract to a collection of hybrid-invariant  $(r + 1)$ -dimensional submanifolds  $M_j \subset D_j$  in finite time.



# Model reduction near hybrid periodic orbit $\gamma$



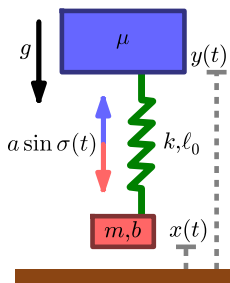
## Corollary (Burden, Revzen, Sastry (arXiv:1308.4158))

*The submanifolds  $M_j$  determine a hybrid system with periodic orbit  $\gamma$ .*

*$\gamma$  is asymptotically stable in the original hybrid system*

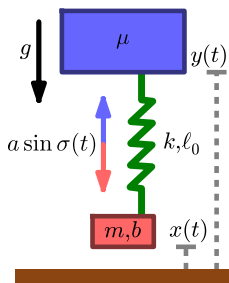
$\iff$   *$\gamma$  is asymptotically stable in the reduced hybrid system.*

# Spontaneous reduction in vertical hopper



Numerically linearizing Poincaré map  $P$  on ground, we find  $DP(\xi)$  has eigenvalue  $\simeq 0.57$ , therefore  $DP^2$  is constant rank near  $\xi$ .

# Spontaneous reduction in vertical hopper

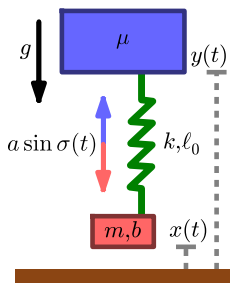


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Corollary (Burden, Revzen, Sastry (arXiv:1308.4158))

hopper reduces one degree-of-freedom after a single “hop”.

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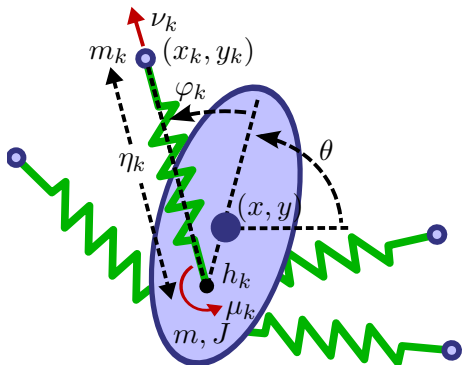
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Interpretation

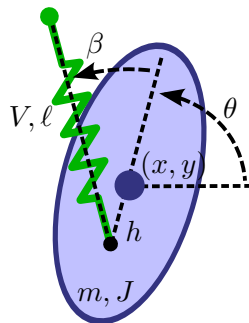
Holonomic ground contact constraint persists after liftoff.



# Model with $n$ legs reduces to Lateral Leg-Spring

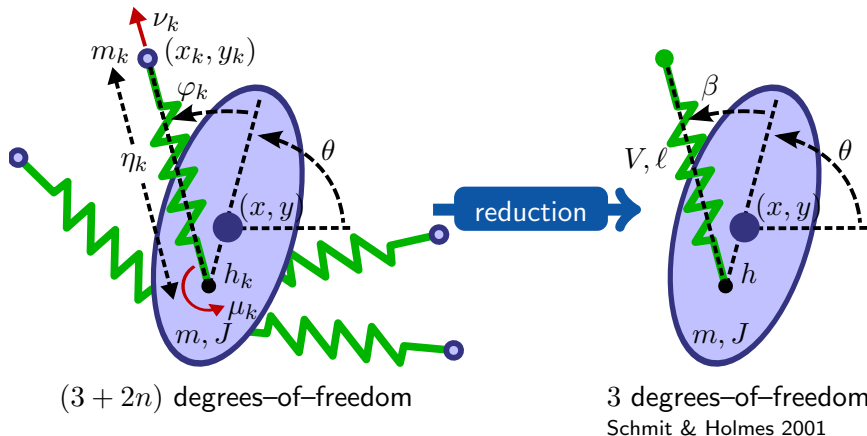


$(3 + 2n)$  degrees-of-freedom



3 degrees-of-freedom  
Schmit & Holmes 2001

# Model with $n$ legs reduces to Lateral Leg-Spring



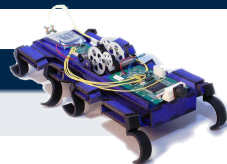
Controller (Burden, Revzen, Sastry (arXiv:1308.4158))

Smooth feedback law reduces  $2n$  degrees-of-freedom after one stride.

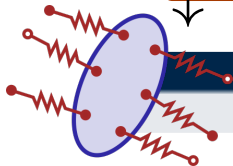
# Parameter Identification



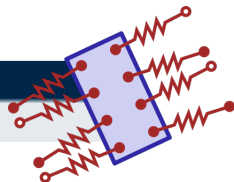
physical system  
animal, robot



2. identification



detailed model  
10–100 DOF



1. reduction



reduced-order model  
< 10 DOF

# Parametric identification for models of rhythmic behavior

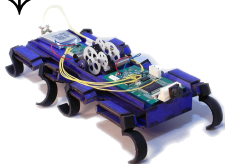


identification

*Periplaneta americana* (Poly-PEDAL Lab, <http://polypedal.berkeley.edu/>)

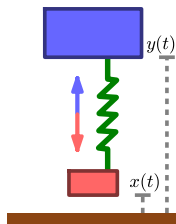


identification

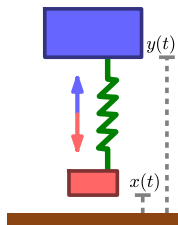


DynaROACH robot (Olin Robotics Lab, <http://orb.olin.edu>)

# Identification of initial conditions

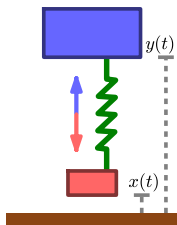


# Identification of initial conditions



$$Y(\phi(t, z)) = y(t)$$

# Identification of initial conditions



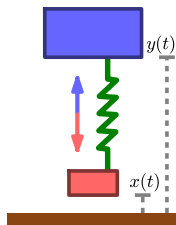
$$Y(\phi(t, z)) = y(t)$$



$$\eta_i = Y(\phi(iT, z^*)) + w_i,$$

$w_i$  iid random variables

# Identification of initial conditions



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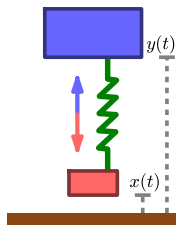
$w_i$  iid random variables

## Identification problem

Solve  $\arg \min_{z \in D_j} \varepsilon(z, \{\eta_i\})$ , where  $\varepsilon(z, \{\eta_i\}) := \sum_i \|Y(\phi(iT, z)) - \eta_i\|^2$ .



# Identification of initial conditions



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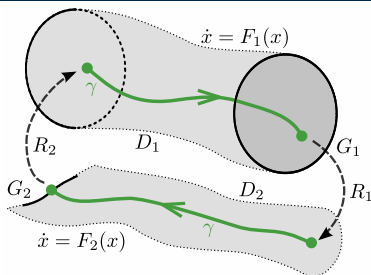
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Solve  $\arg \min_{z \in D_j} \varepsilon(z, \{\eta_i\})$ , where  $\varepsilon(z, \{\eta_i\}) := \sum_i \|Y(\phi(iT, z)) - \eta_i\|^2$ .

## Assumption (smooth observations)

$Y$  is smooth along trajectories, i.e.  $Y(\phi(t, z))$  is a smooth function of  $t$ .

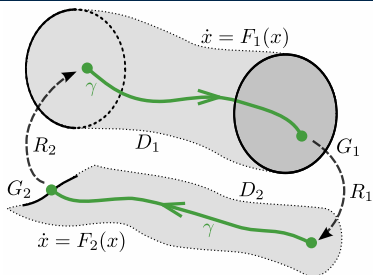
# Identification on original hybrid model vs. reduced model



Identification on  $\bigcup_j D_j$

$$\arg \min_{z \in D_j} \varepsilon(z, \{\eta_i\})$$

# Identification on original hybrid model vs. reduced model



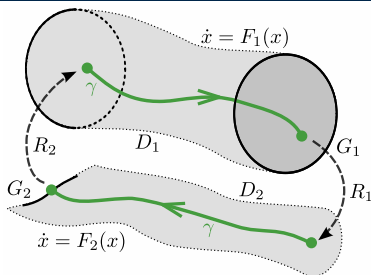
Identification on  $\bigcup_j D_j$

$$\arg \min_{z \in D_j} \varepsilon(z, \{\eta_i\})$$

$\nabla \varepsilon$  undefined on  $G_j \subset D_j$

$R_j$  not generally invertible

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Identification on  $\bigcup_j D_j$

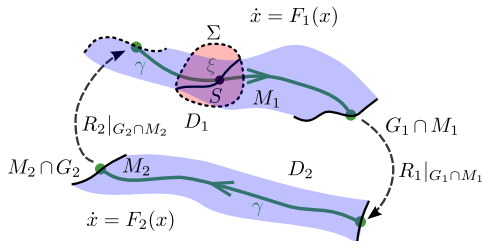
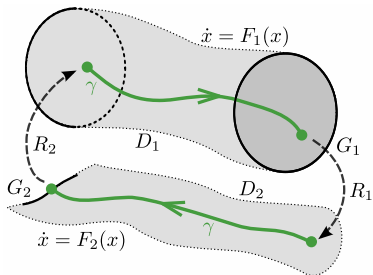
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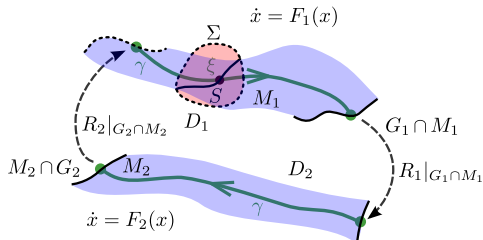
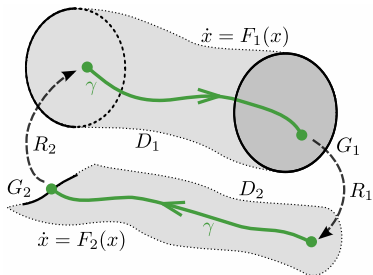
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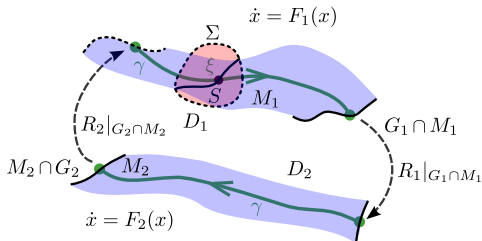
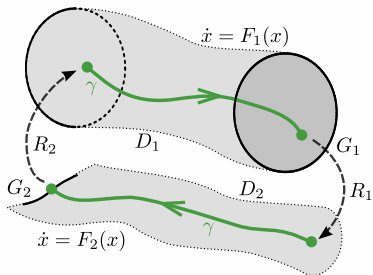
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$R_j|_{M_j}$  invertible

**first-order algorithms** applicable

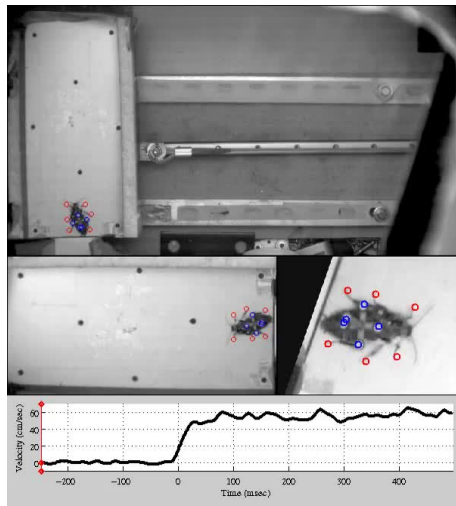
# Novel quantitative predictions for biomechanics



observation

neural feedback  
appears at a delay

Revzen, Burden et al. BC 2013





# Novel quantitative predictions for biomechanics



observation

neural feedback  
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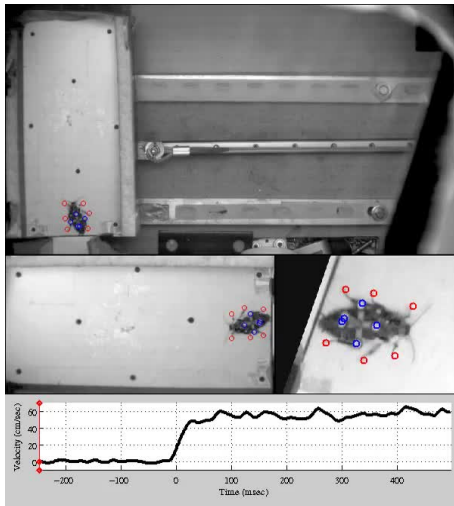
Revzen, Burden et al. BC 2013

identification

prediction

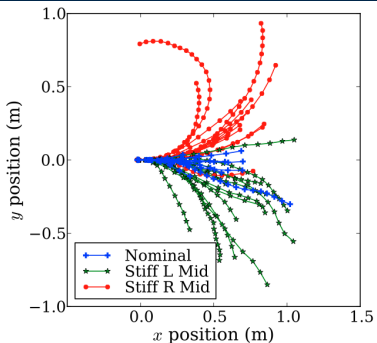
passive mechanics  
sensitive to inertia

Full et al. 2002



Burden, Revzen, Moore, Sastry, Full SICB 2013

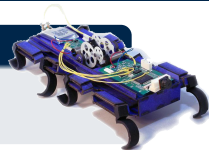
# Model-based design and control of dynamic robots



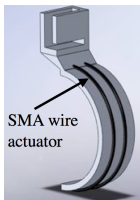
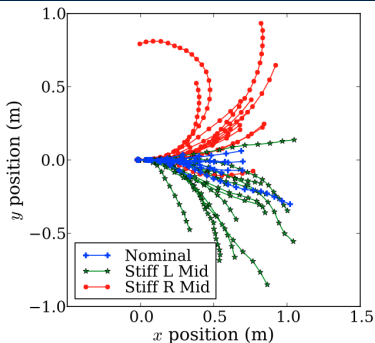
design

minimal use  
of actuators

Hoover et al. 2010



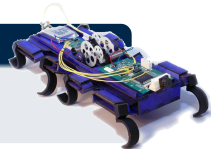
# Model-based design and control of dynamic robots



design

minimal use  
of actuators

Hoover et al. 2010



identification

control

asymmetric leg  
stiffness change

Proctor & Holmes 2008

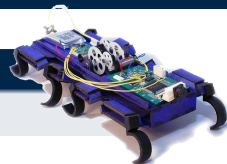


# Convergent Simulation



physical system

animal, robot



identification

detailed model

10–100 DOF

reduction

simple model

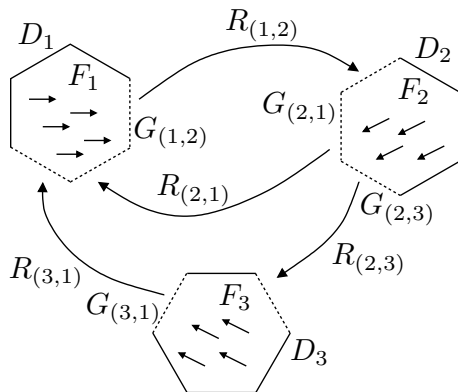
< 10 DOF

simulation

# State space metric

Hybrid control systems comprised of distinct operating “modes”

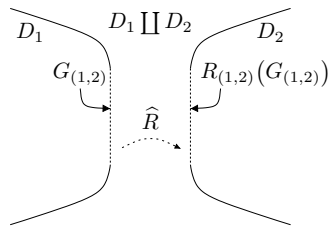
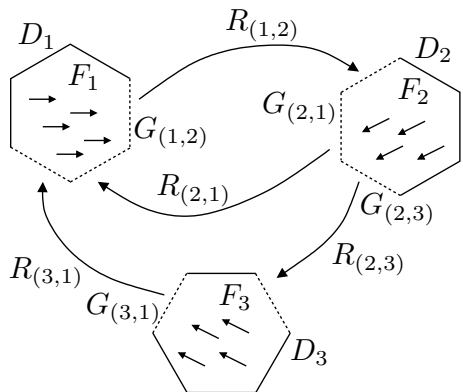
- Digital controller state (“on” or “off”)
- Physical/dynamical regime (“reach” or “grasp”)



# State space metric

Hybrid control systems comprised of distinct operating “modes”

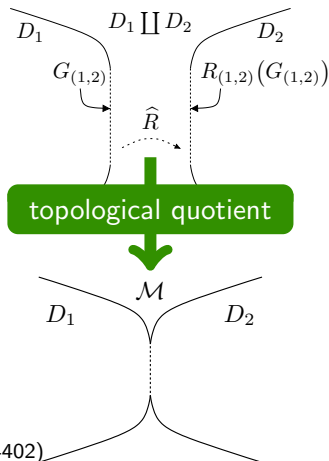
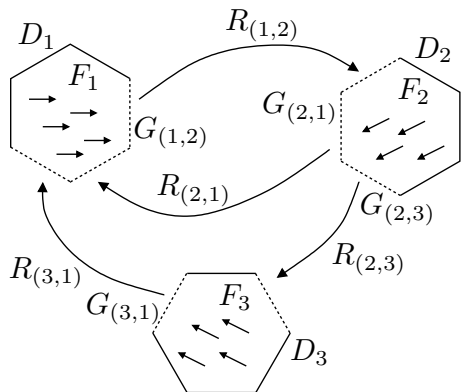
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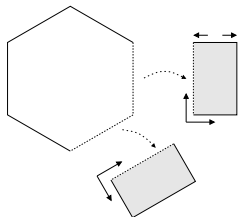
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# Convergent numerical simulation

Transition between discrete modes occurs autonomously

- simulation algorithm must control error introduced by “event detection”

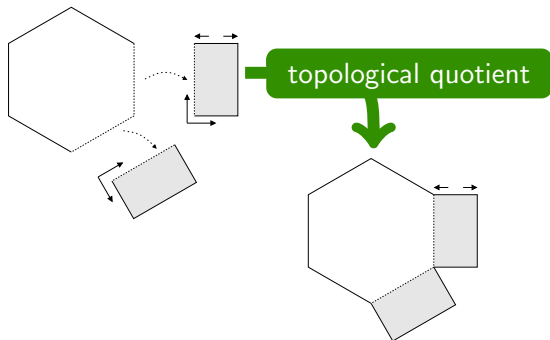




# Convergent numerical simulation

Transition between discrete modes occurs autonomously

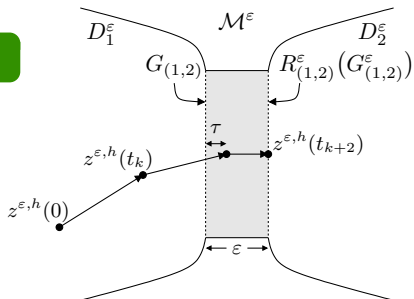
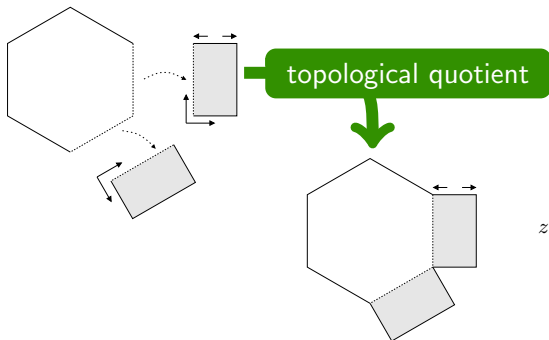
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# Convergent numerical simulation

Transition between discrete modes occurs autonomously

- simulation algorithm must control error introduced by “event detection”



State space metric enables proof of convergence for “Forward Euler”

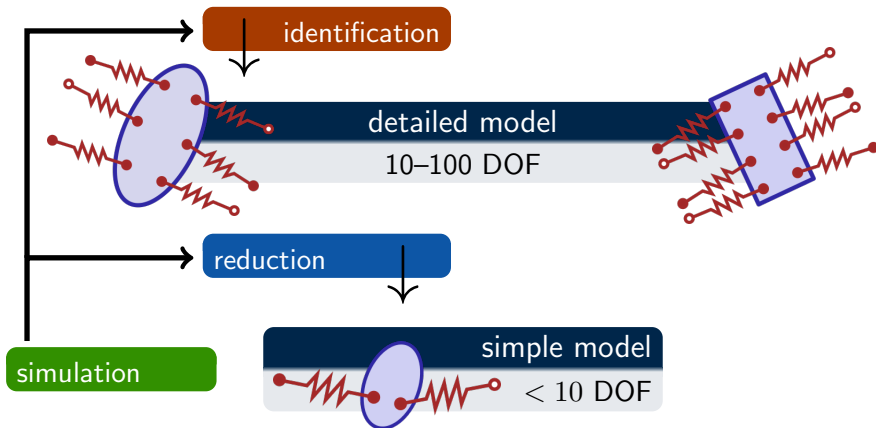
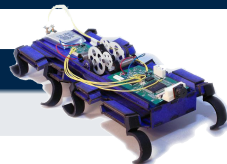
- Linear convergence rate for any orbitally stable execution

# Models enable translation across scale and morphology

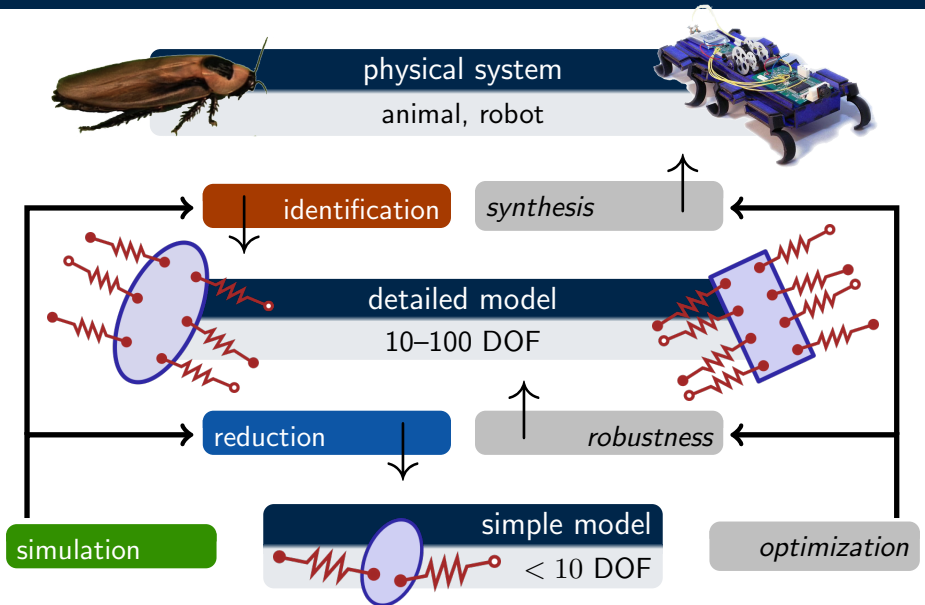


physical system

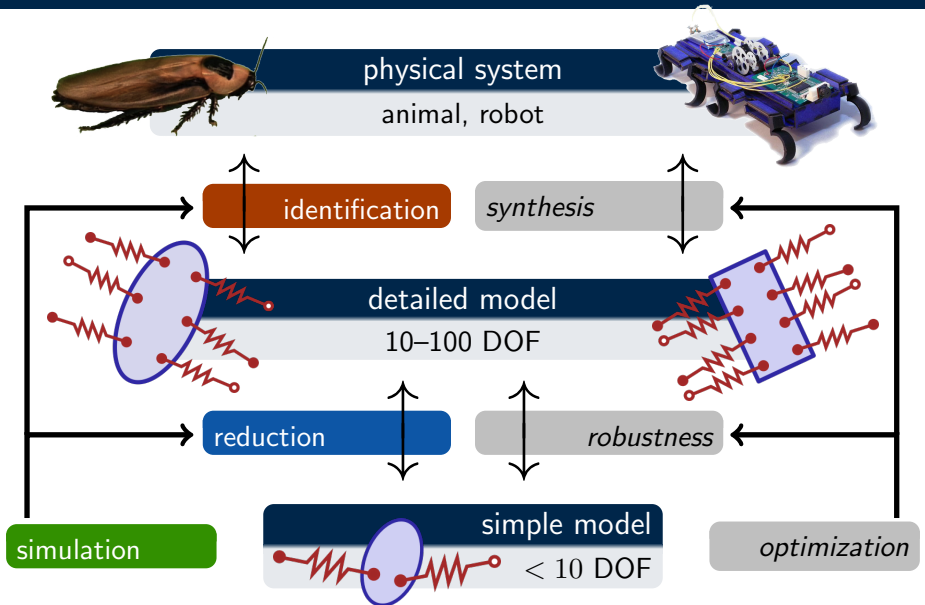
animal, robot



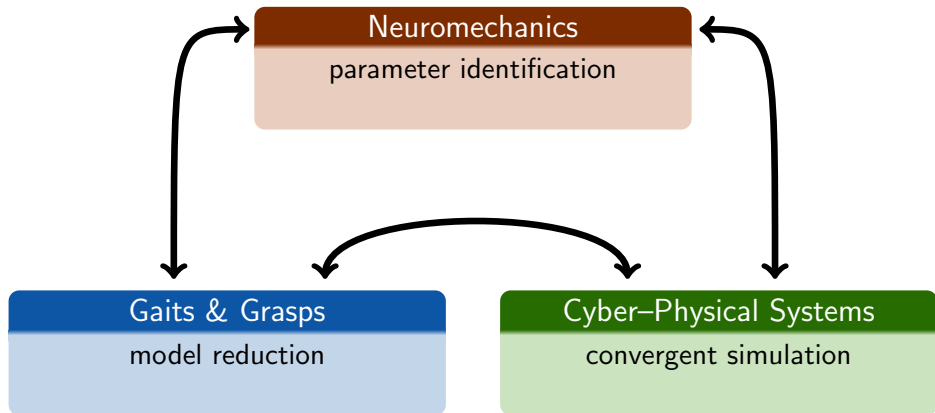
# Models enable translation across scale and morphology



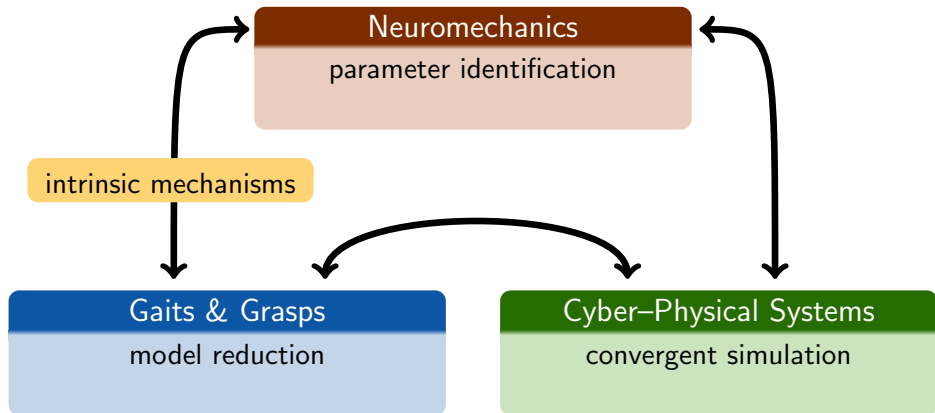
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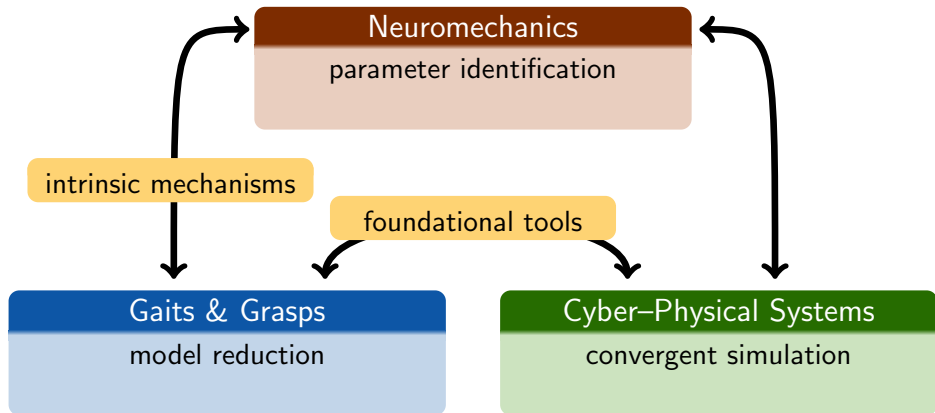
# Summary of framework for studying locomotion



# Summary of framework for studying locomotion

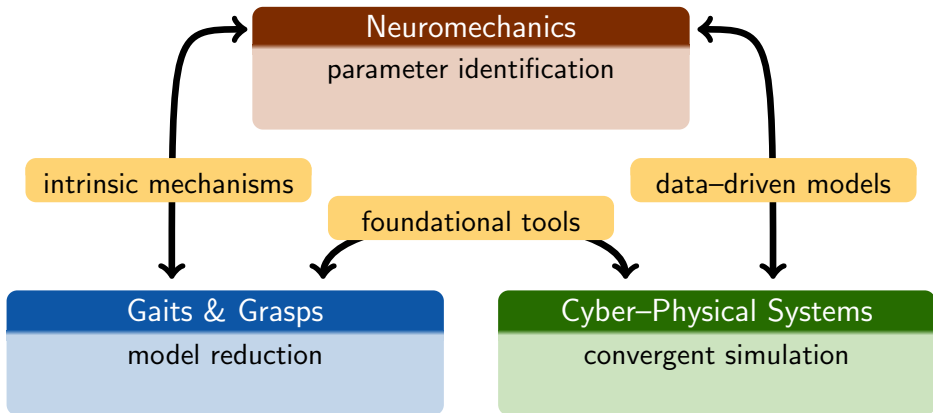


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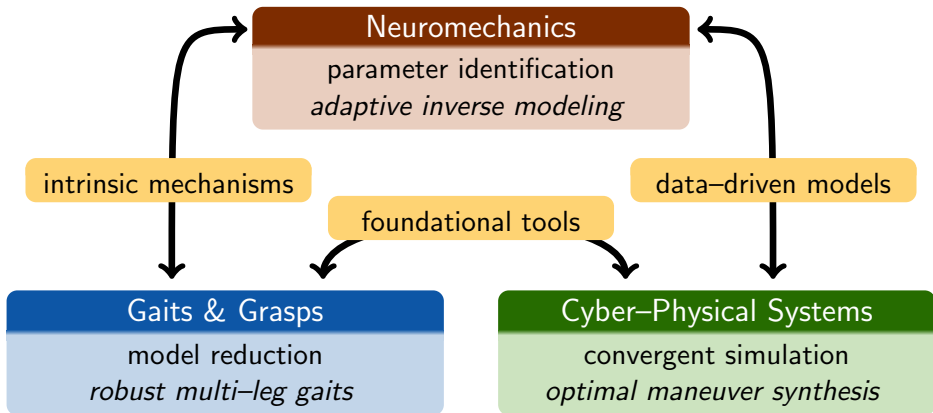




# Summary of framework for studying locomotion



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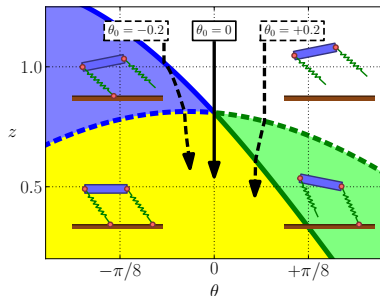
# Robust multi-legged gaits



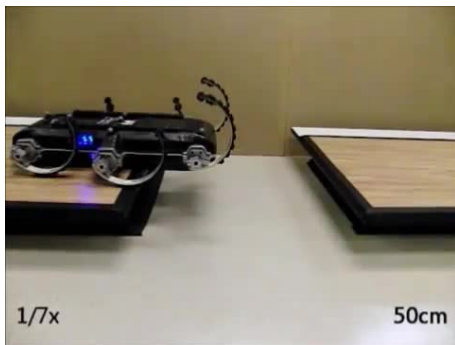
U. Minnesota Equine Center



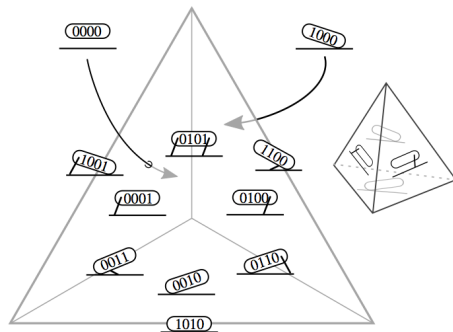
[www.naturhov.dk](http://www.naturhov.dk)



# Optimal maneuver synthesis



X-RHex Lite (<http://kodlab.seas.upenn.edu>)

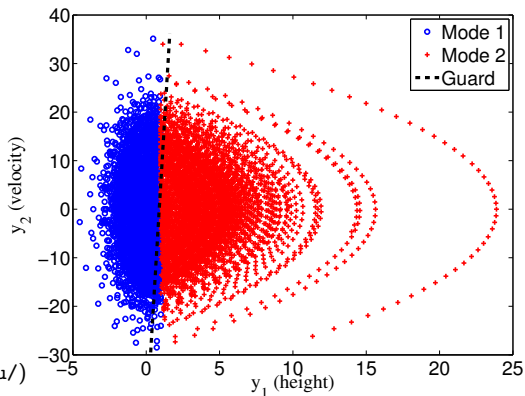
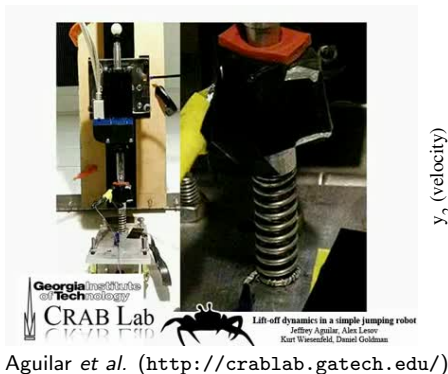


Johnson & Koditschek ICRA 2013

Reformulate combinatorial problem

Control yields footfall sequence; can search over continuous inputs.

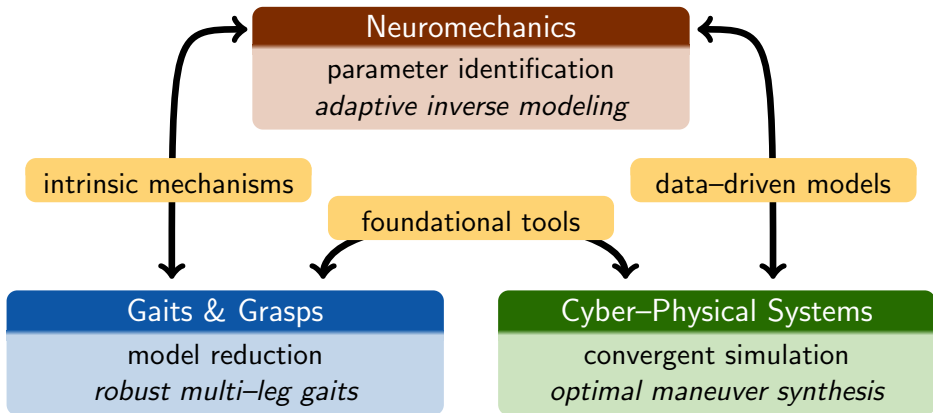
# Adaptive inverse modeling

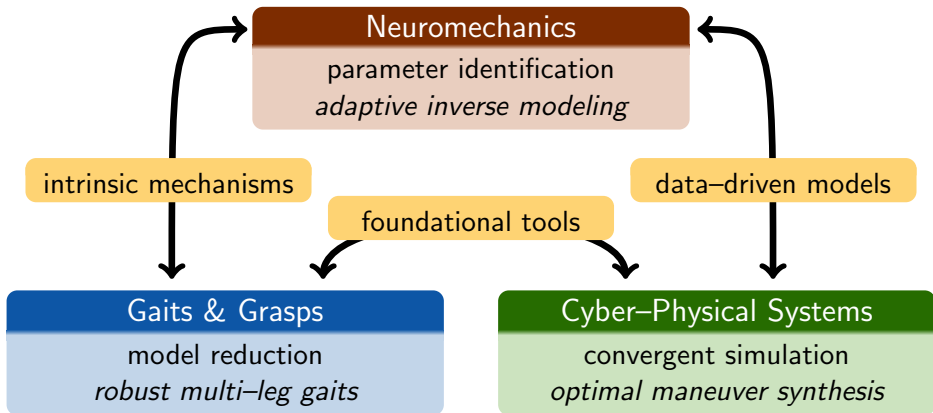


Estimate piecewise-affine model from empirical data

Use predictive model for controller synthesis

Elhamifar, Burden, Sastry (submitted)





## An emerging Systems Theory for Neuromechanics

Engineer dynamic interactions between computation & mechanics

# Discussion & Questions — Thanks for your time!

## Reduction

Reduced-order model emerges from intermittent contact.

## Simulation

Convergent numerical simulation for hybrid control systems.

## Identification

Scalable parameter identification for models of locomotion.

## Collaborators

- Shankar Sastry (UCB)
- Robert Full (UCB)
- Dan Koditschek (UPenn)
- Shai Revzen (UMich)
- Aaron Hoover (Olin)
- Henrik Ohlsson (Linköping)

## Funding

- NSF Fellowship
- ARL MAST CTA (W911NF-08-2-0004)

