Hybrid Models for Dynamic and Dexterous Robots

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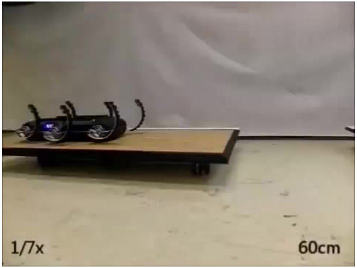
October 24, 2014

Dynamic and dexterous robots

Hodgins & Raibert IJRR 1990

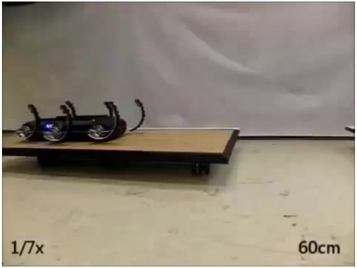


Johnson & Koditschek ICRA 2013

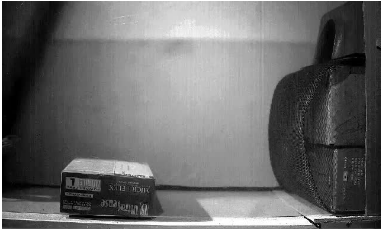
Dynamic and dexterous robots vs. animals

Hodgins & Raibert IJRR 1990

Bill Roth 1996 US Gymnastics Championship



Johnson & Koditschek ICRA 2013



Libby, Moore, Chang–Siu, Li, Cohen, Jusufi, Full Nature 2012

Locomotion, manipulation arise from intermittent contact

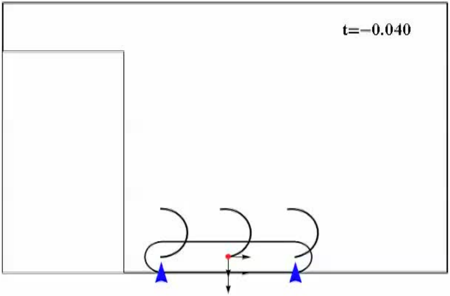
Johnson & Koditschek ICRA 2013

Senoo, Yamakawa, Mizusawa, Namiki, Ishikawa, Shimojo IROS 2009

Li, Hsieh, Goldman JEB 2012

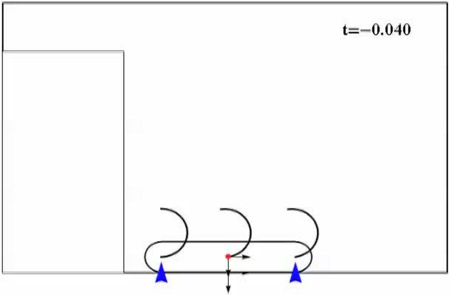
Parsimonious models for intermittent contact

Johnson & Koditschek ICRA 2013



Parsimonious models for intermittent contact

Johnson & Koditschek ICRA 2013



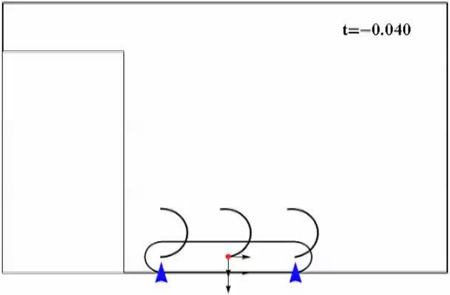
Dynamics with $n \in \mathbb{N}$ limbs, intrinsic coordinates $q \in Q$

- Each subset of contact limbs $J \subset \{1, \ldots, n\}$ determine continuous dynamics $\ddot{q} = f(q, \dot{q}) + \lambda_J(q, \dot{q}) Da_J(q)$ subject to constraints $a_J(q) \equiv 0$.
- At impact into mode J, velocities update discontinuously: $\dot{q}^+ = \Delta_J \dot{q}^-$.

Johnson, Burden, Koditschek (*in prep*)
A Hybrid Systems Model for Simple Manipulation and Self–Manipulation Systems

Parsimonious models for intermittent contact

Johnson & Koditschek ICRA 2013



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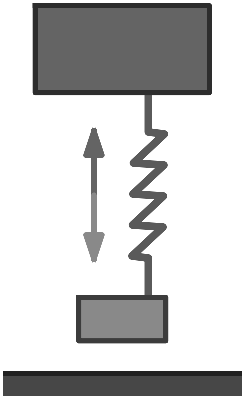
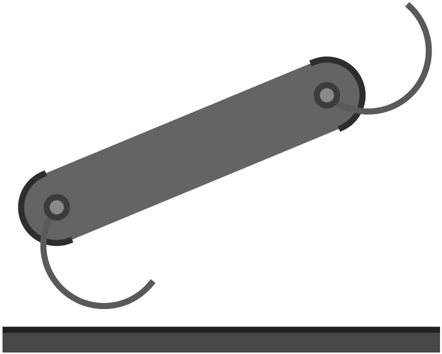
Yields a piecewise–defined ("hybrid") model for (self–)manipulation.

Johnson, Burden, Koditschek (*in prep*)
A Hybrid Systems Model for Simple Manipulation and Self–Manipulation Systems

Pathologies in hybrid models for intermittent contact





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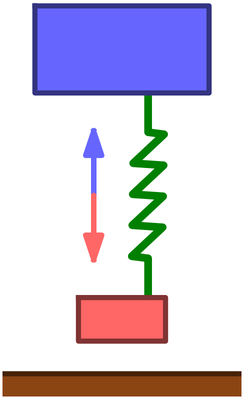
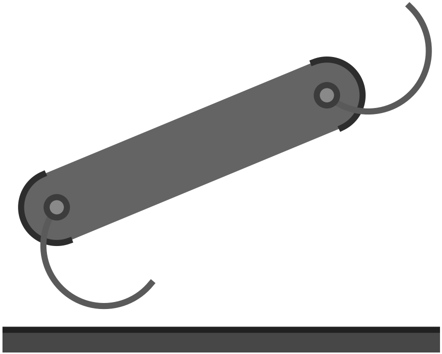
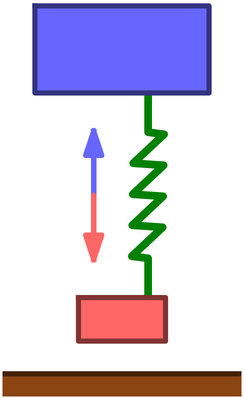


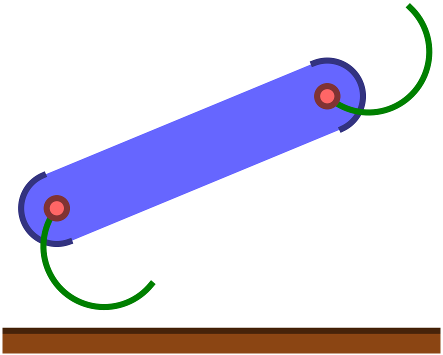


1. Discontinuities

equations–of–motion and states change abruptly at impact

Pathologies in hybrid models for intermittent contact





1. Discontinuities

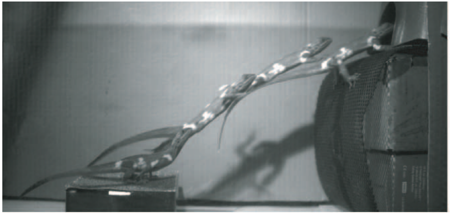
equations–of–motion and states change abruptly at impact

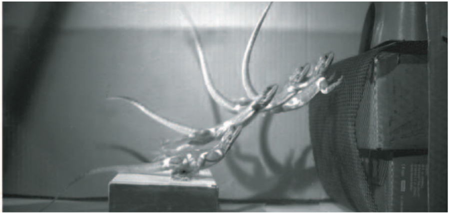
2. Inconsistencies

restitution laws lead to nondeterminism at impact

Pathologies are not "natural"

Libby, Moore, Chang–Siu, Li, Cohen, Jusufi, Full Nature 2012





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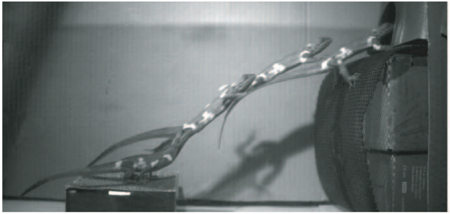

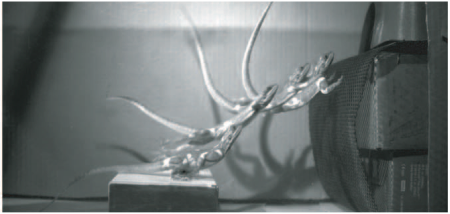

Mathematical models approximate the physical world

Pathologies indicate bad models or deficient analysis.

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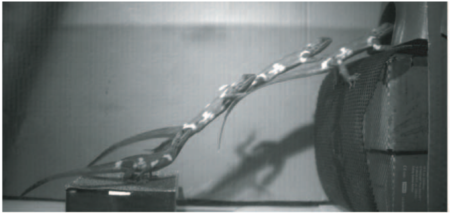

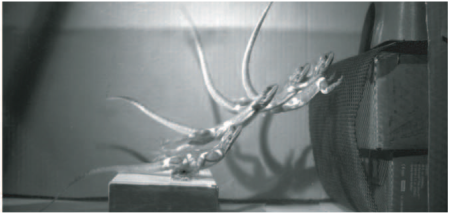

Mathematical models approximate the physical world

Pathologies indicate bad models or deficient analysis.

1. Remove discontinuities

construct intrinsic state space that removes discontinuities

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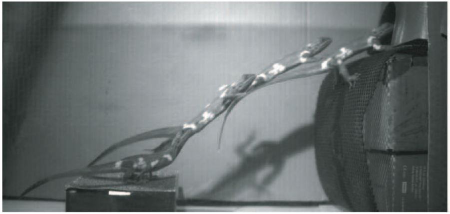
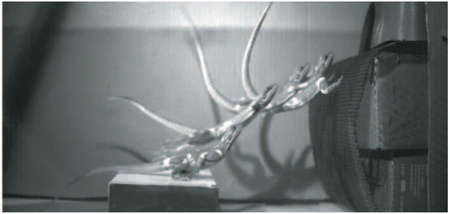




Mathematical models approximate the physical world

Pathologies indicate bad models or deficient analysis.

1. Remove discontinuities

construct intrinsic state space that removes discontinuities

2. Resolve inconsistencies

restrict restitution laws to obtain piecewise–differentiable flow

Outline

Motivation: animals possess rich behavioral repertoire robots lack

Progress hampered by pathologies in parsimonious models.

1. Topological quotient removes discontinuities

   Enables convergent numerical simulation for legged locomotion.

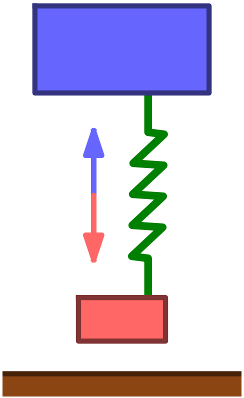
2. Restricting impact restitution resolves inconsistencies

   Enables scalable nonsmooth optimization and control of locomotion.

Future directions: towards sensorimotor control theory

Synthesis and stabilization of rhythmic behaviors, aperiodic maneuvers.

Hybrid models for dynamic and dexterous robots



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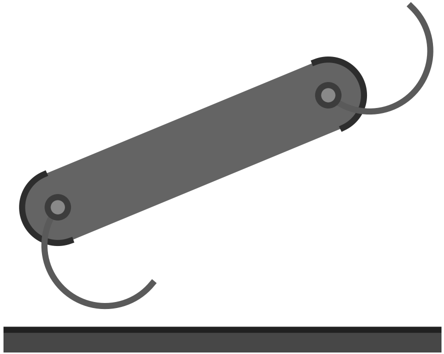
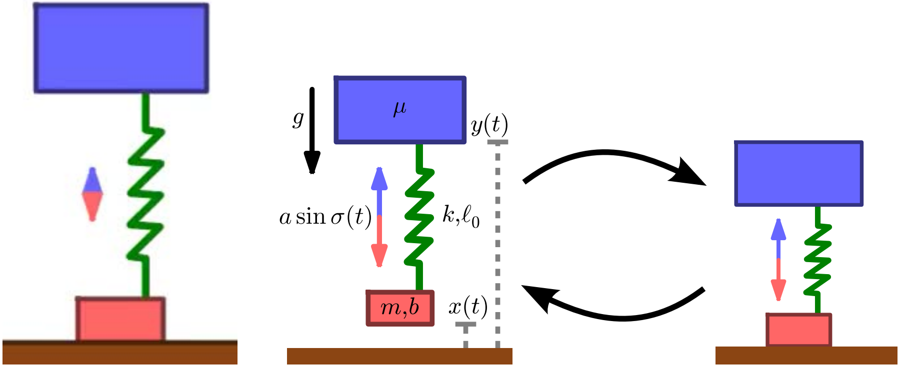


2. Resolve inconsistencies

Discontinuities in vertical hopping



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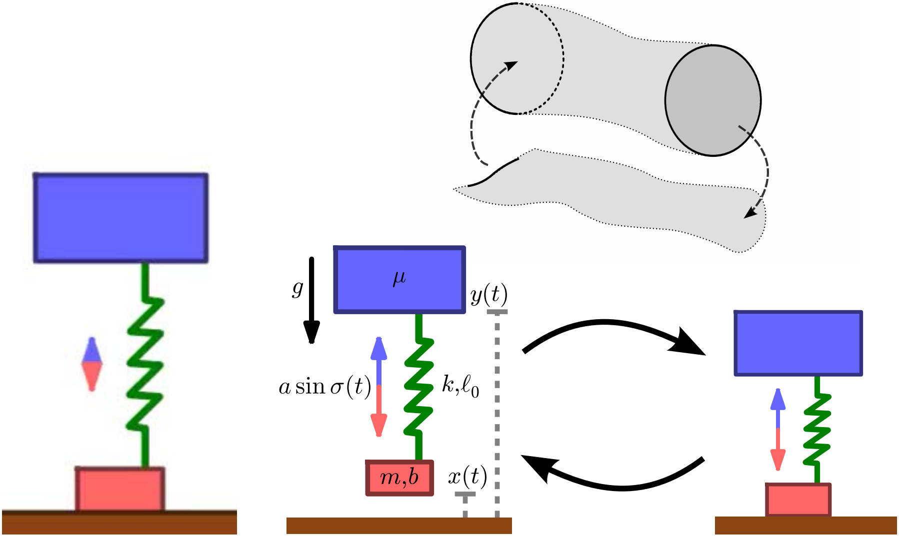
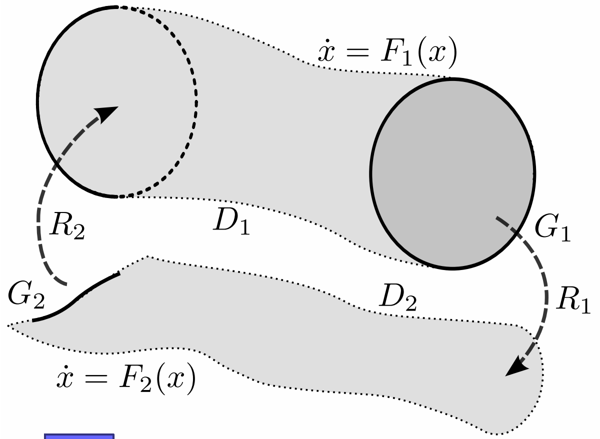
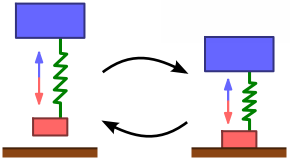
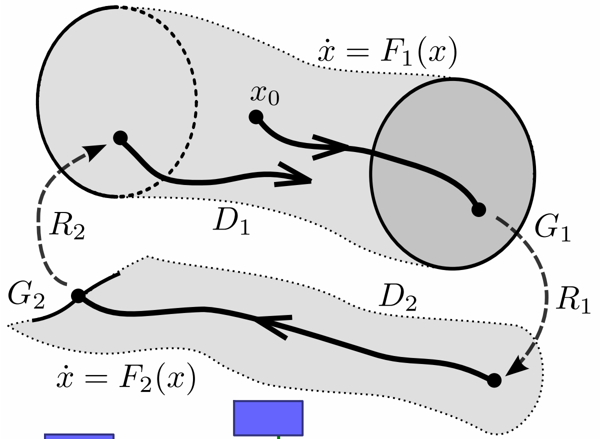


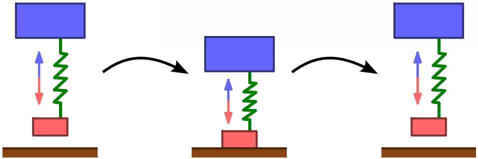
Hybrid dynamical system





Trajectory for a hybrid dynamical system





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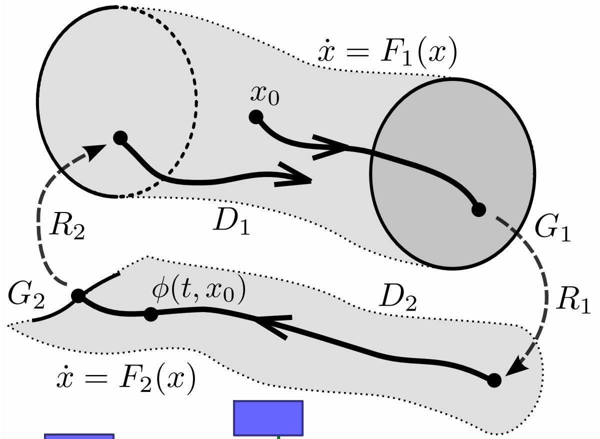
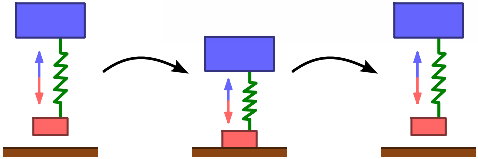
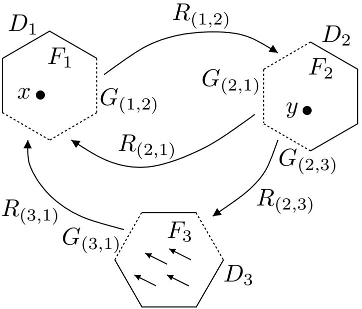




Distance metric and simulation algorithm

Hybrid control systems comprised of distinct operating "modes"

- Digital controller state ("on" or "off")
- Physical/dynamical regime ("reach" or "grasp")



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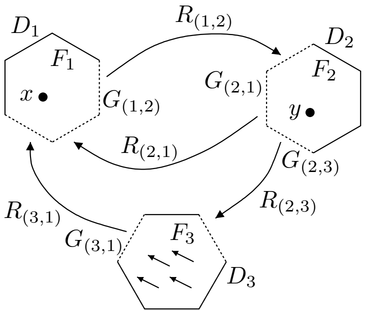

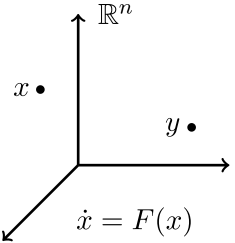

Classical ODE system

- distance: $d(x, y) = \|x - y\|$
- simulation: $x_{k+1} = x_k + hF(x_k)$

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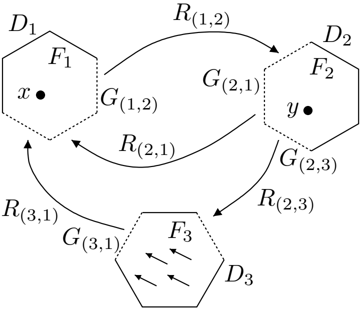
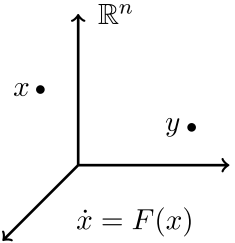




Hybrid dynamical system

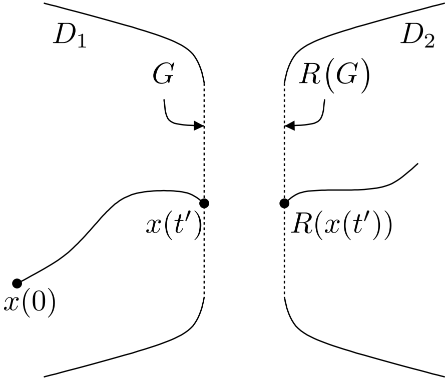
- distance: $d(x, y) = \infty$
- simulation: $x_k + hF(x_k) \notin D$

Classical ODE system

- distance: $d(x, y) = \|x - y\|$
- simulation: $x_{k+1} = x_k + hF(x_k)$

Remove discontinuities via topological quotient

disjoint state space $D_1 \coprod D_2$



Remove discontinuities via topological quotient

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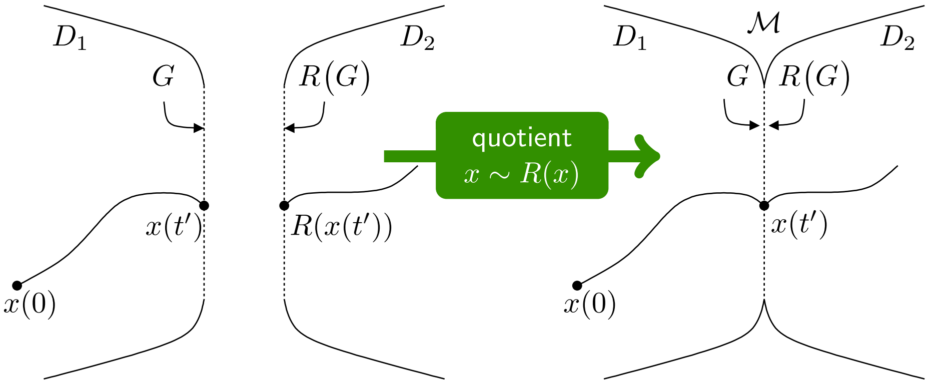
quotient space $\mathcal{M}$



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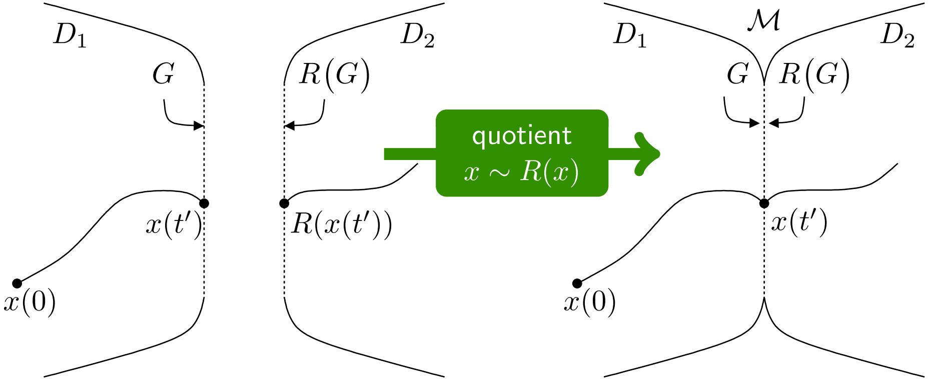
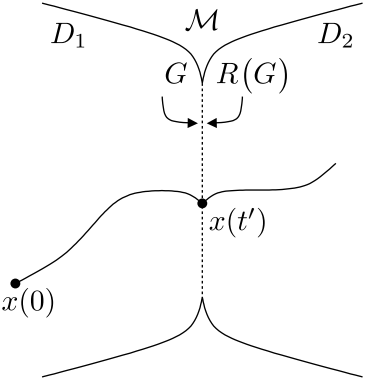


Theorem (arXiv:1302.4402)

$\mathcal{M}$ *is metrizable.*

Relax quotient space at discrete transitions

quotient space $\mathcal{M}$



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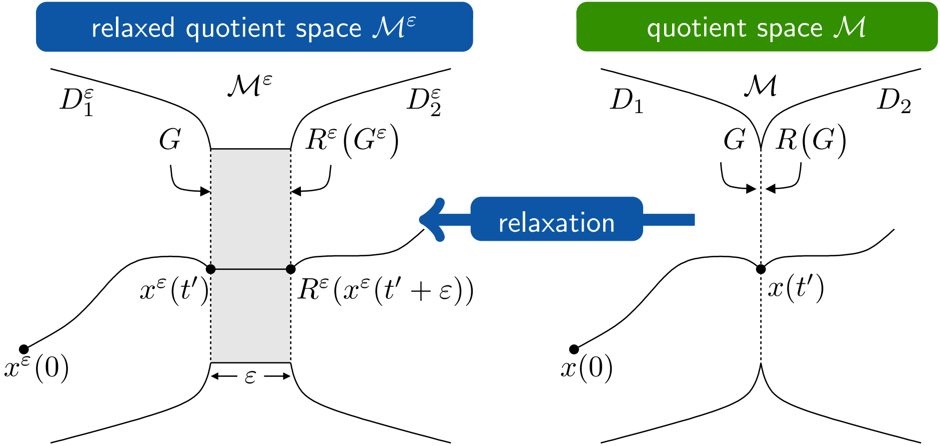
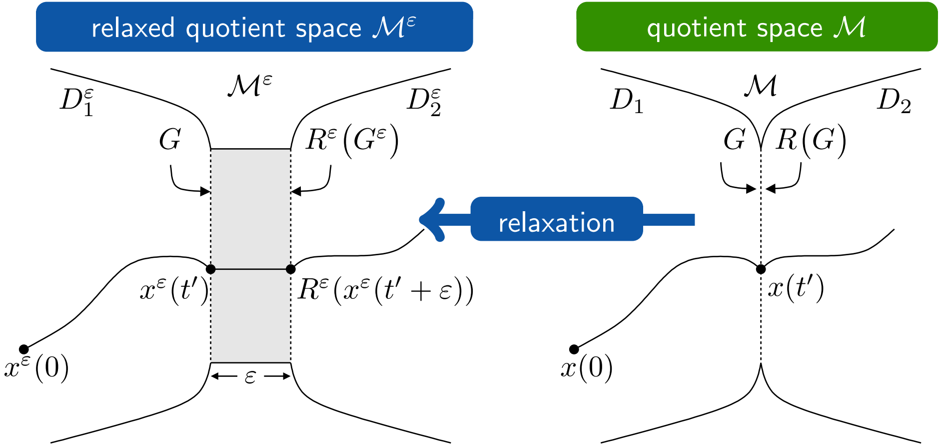


Relax quotient space at discrete transitions



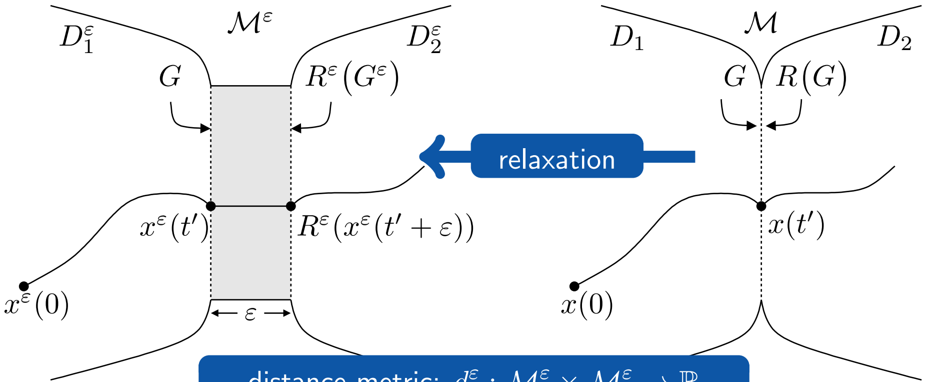
Theorem (arXiv:1302.4402)

$\mathcal{M}^\varepsilon$ is metrizable.

Relax quotient space at discrete transitions

relaxed quotient space $\mathcal{M}^\varepsilon$

quotient space $\mathcal{M}$



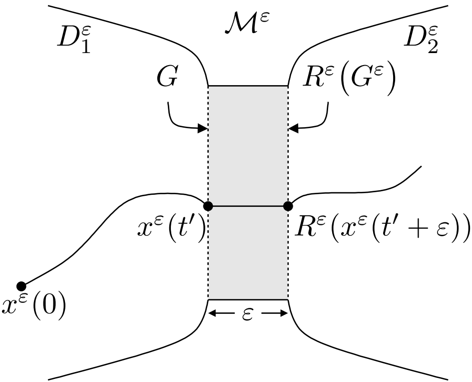
distance metric: $d^\varepsilon : \mathcal{M}^\varepsilon \times \mathcal{M}^\varepsilon \to \mathbb{R}$

Theorem (arXiv:1302.4402)

$\mathcal{M}^\varepsilon$ is metrizable.

Numerical simulation on relaxed quotient space

relaxed execution x^ε



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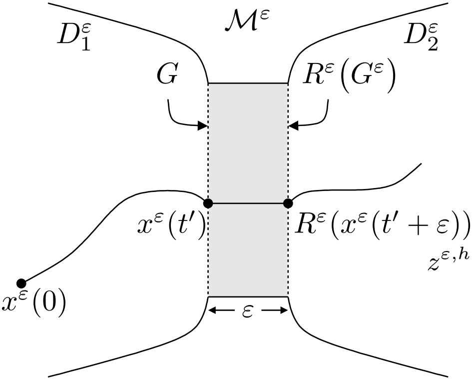
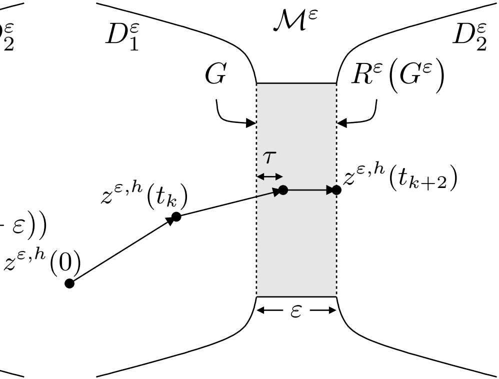


discrete approximation $z^{\varepsilon,h}$



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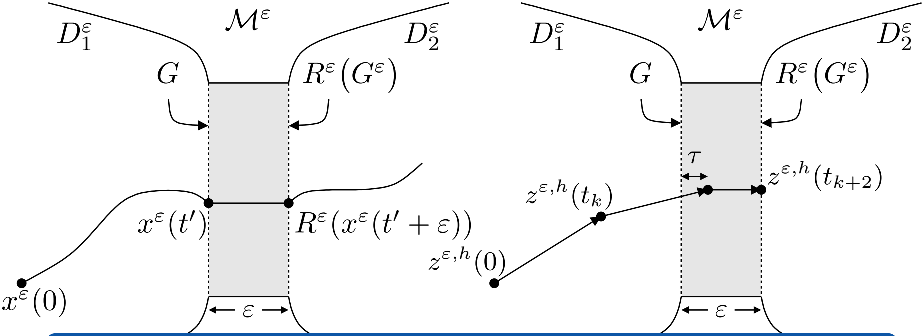

trajectory metric: $\rho^\varepsilon(x, z^{\varepsilon,h}) = \sup\left\{d^\varepsilon(x^\varepsilon(s), z^{\varepsilon,h}(s)) : s \in [0,t]\right\}$

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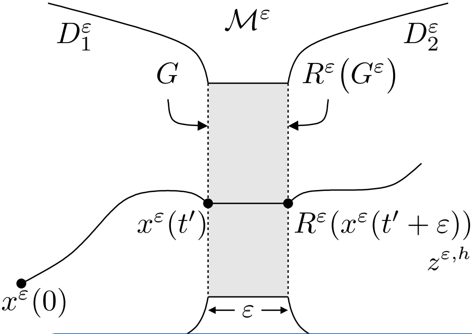

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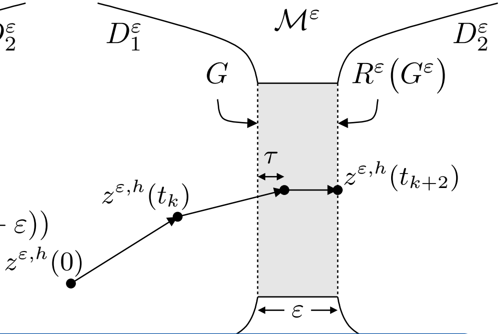

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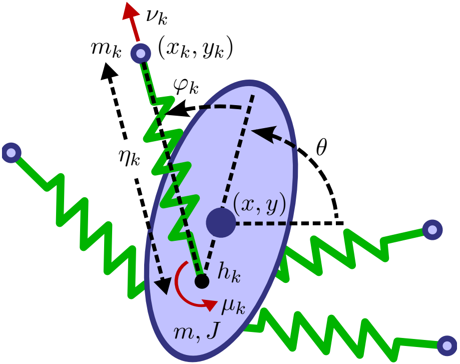
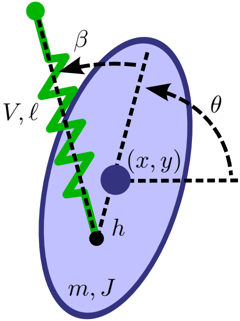
Theorem (arXiv:1302.4402)

If x is orbitally stable then $\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) \in O(\varepsilon) + O(h)$.

Implication for controlling dynamic and dexterous robots



$(3 + 2n)$ degrees–of–freedom



3 degrees–of–freedom
Schmit & Holmes 2001

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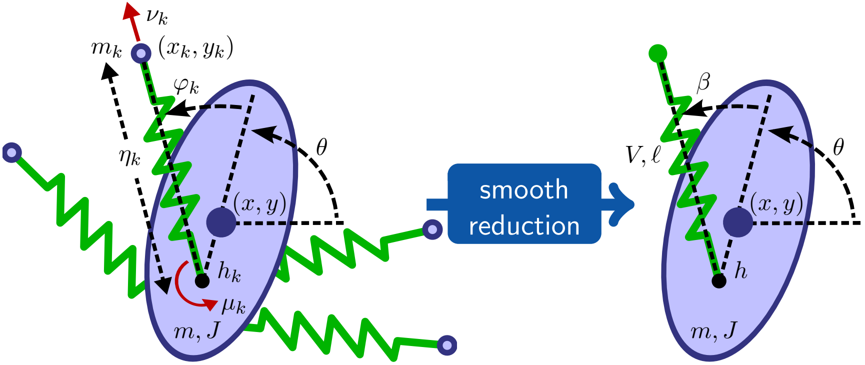

$(3 + 2n)$ degrees–of–freedom

3 degrees–of–freedom
Schmit & Holmes 2001

Controlled reduction (arXiv:1308.4158)

Smooth feedback law reduces $2n$ degrees–of–freedom after one stride.

Contribution from removal of discontinuities

Motivation: animals possess rich behavioral repertoire robots lack

Progress hampered by pathologies in parsimonious models.

1. Topological quotient removes discontinuities

   Enables convergent numerical simulation for legged locomotion.

2. Restricting impact restitution resolves inconsistencies

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Future directions: towards sensorimotor control theory

Synthesis and stabilization of rhythmic behaviors, aperiodic maneuvers.

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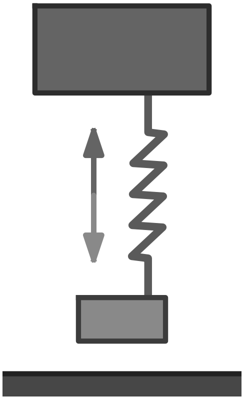
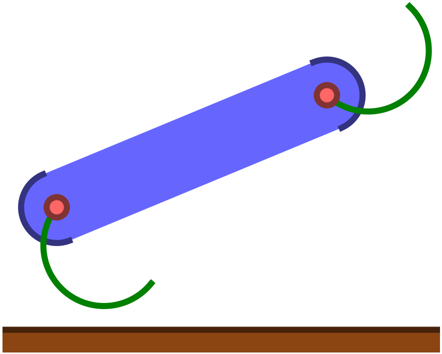


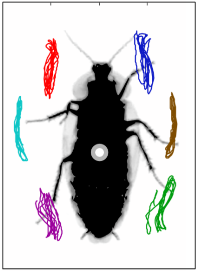


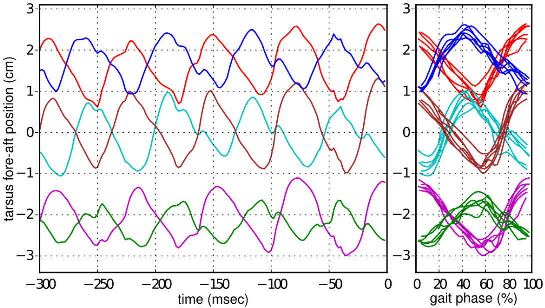
1. Remove discontinuities

2. Resolve inconsistencies

Near–simultaneous limb touchdown in animal gaits

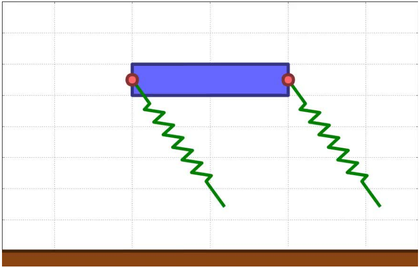
alternating tripod





trot

MeMyHorseAndI.com

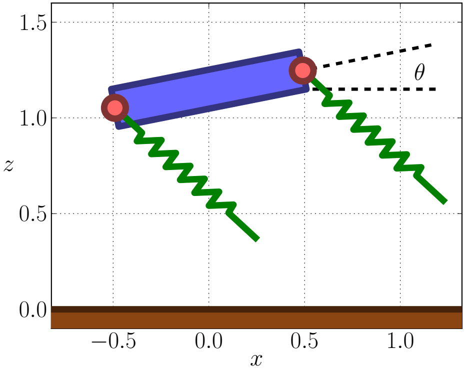


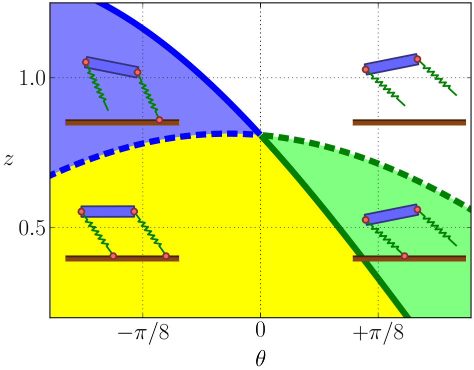
Near–simultaneous limb touchdown in robot gaits

Galloway, Haynes, Ilhan, Johnson, Knopf, Lynch, Plotnick, White, Koditschek UPenn 2010

Hyun, Seok, Lee, Kim IJRR 2014

Dynamics of near–simultaneous limb touchdown





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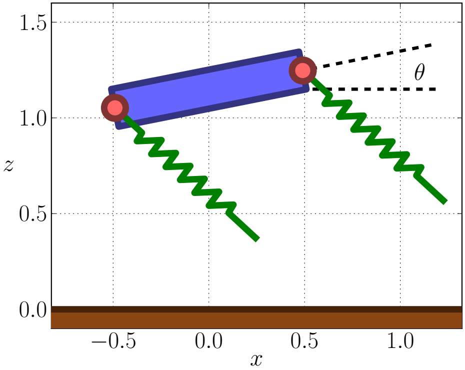

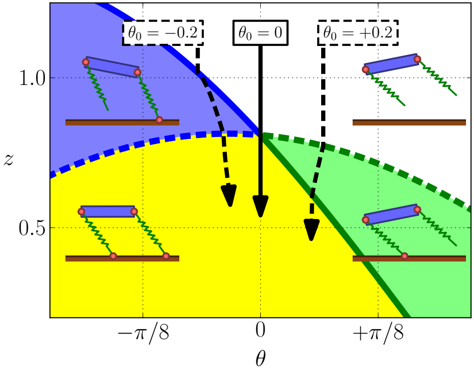

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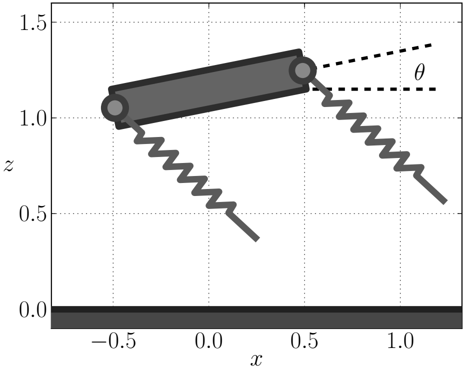

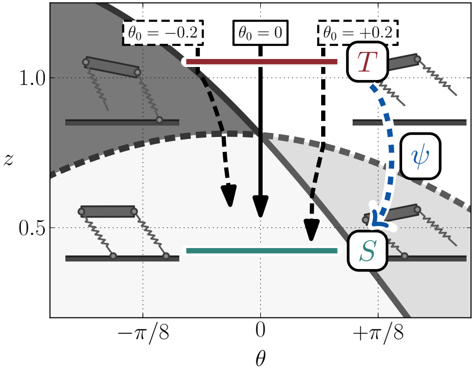

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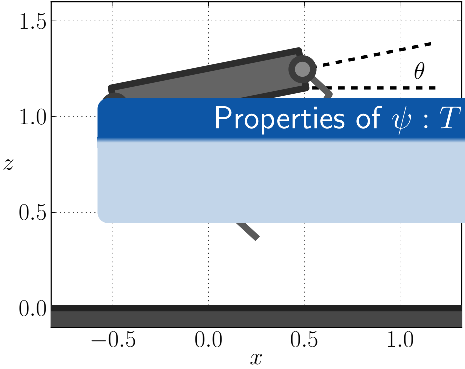
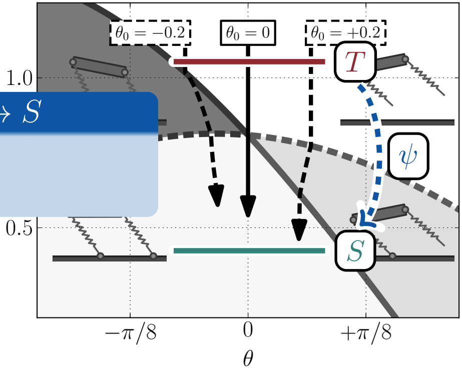


Properties of $\psi : T \to S$



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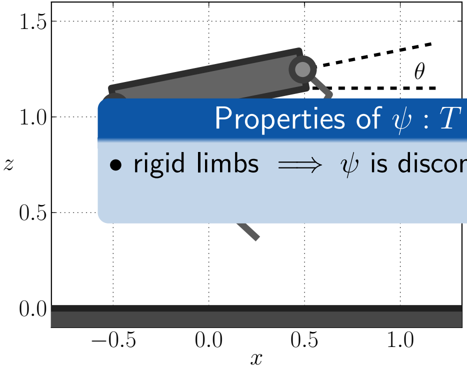
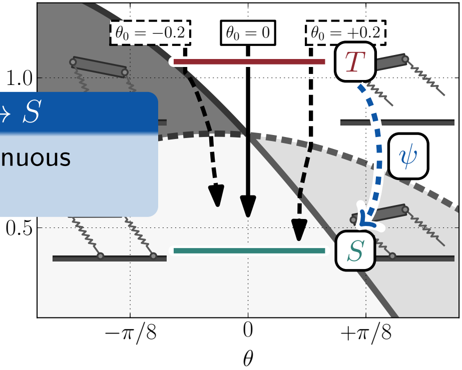
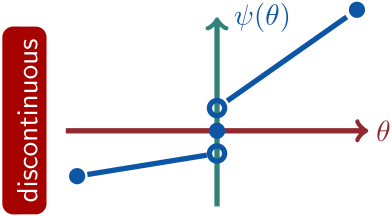


Properties of $\psi : T \to S$

- rigid limbs $\implies$ ψ is discontinuous





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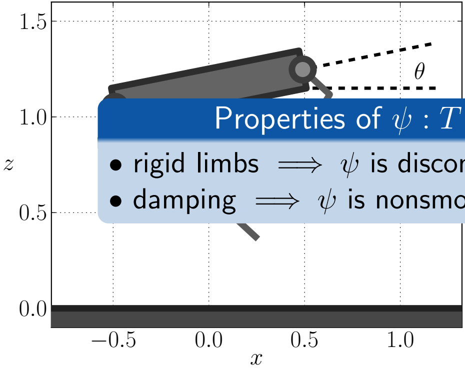
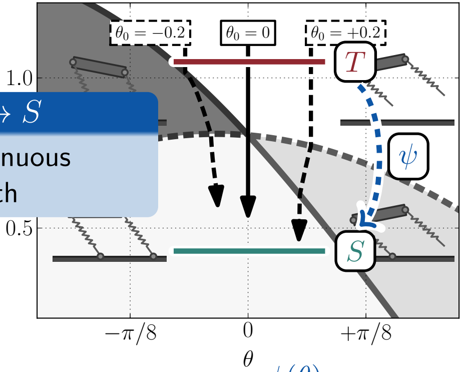
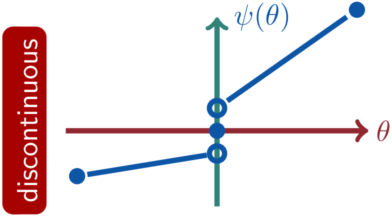
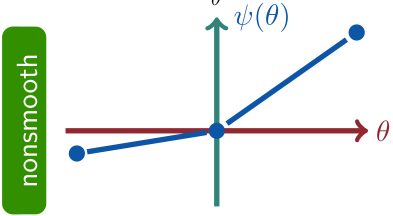




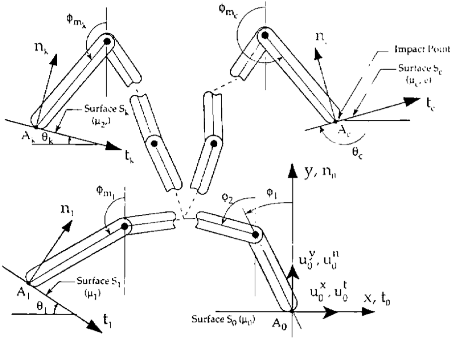
Properties of $\psi : T \to S$

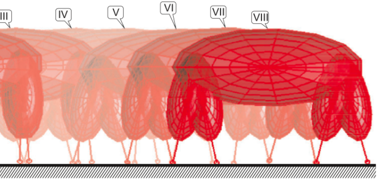
- rigid limbs $\implies$ ψ is discontinuous
- damping $\implies$ ψ is nonsmooth





Rigidity leads to inconsistencies at impact





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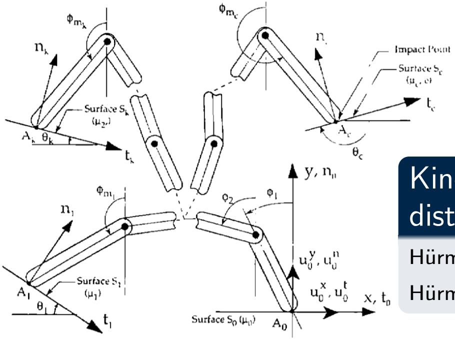

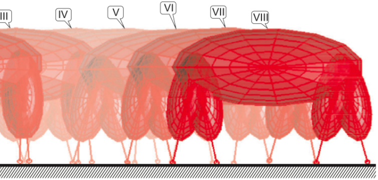

Kinematic hands admit 5 (!) distinct outcomes after grasp

Hürmüzlü and Marghitu IJRR 1994

Hürmüzlü and Marghitu JAM 1995

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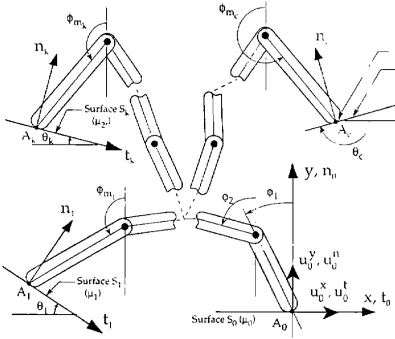

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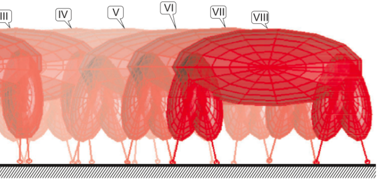
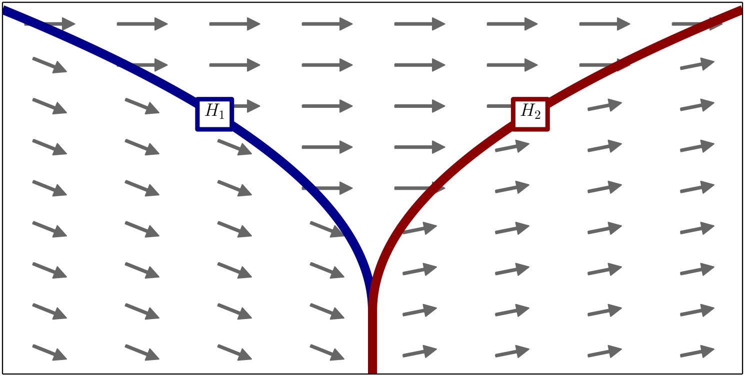


Quadruped model possesses three distinct trot gaits

Remy, Buffington, Siegwart IJRR 2010

Damping leads to nonsmooth flow through impact



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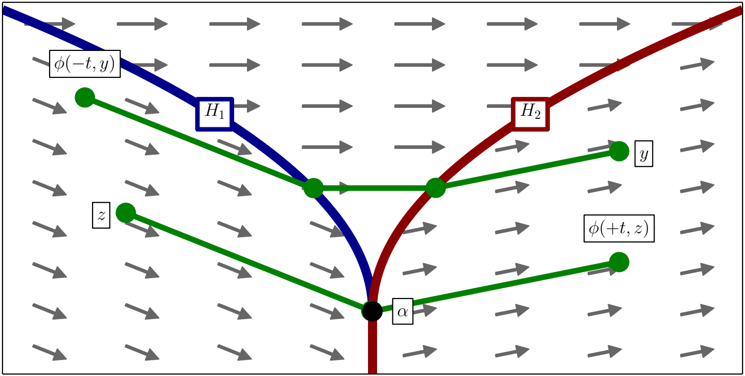

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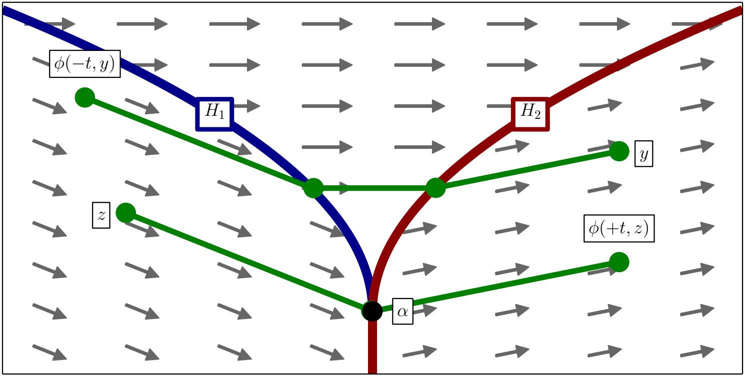

Theorem (arXiv:1407.1775)

Discontinuous vector field $\dot{x} = F(x)$ *yields nonsmooth flow* $\phi : \mathcal{F} \to \mathbb{R}$*:*

$$\forall (t,x) \in \mathcal{F} \subset \mathbb{R} \times \mathbb{R}^d : \phi(t,x) = x + \int_0^t F(\phi(s,x))\, ds.$$

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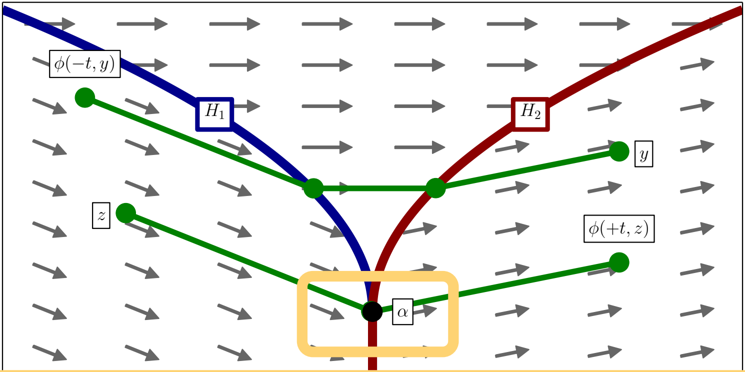

ϕ is nonsmooth since $D_t\phi$ is undefined e.g. at $\alpha \in H_1 \cap H_2$

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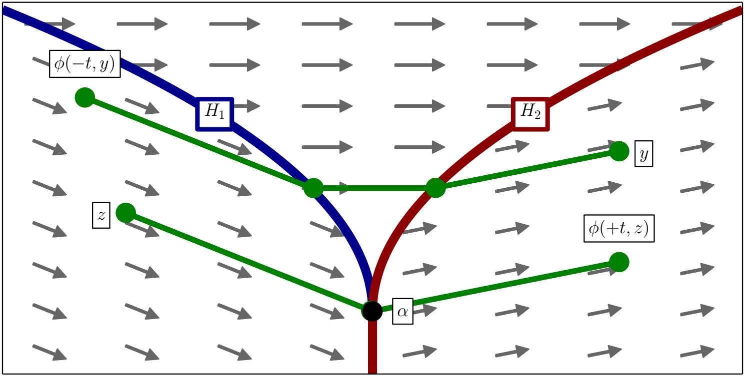
Nonsmooth flow $\phi : \mathcal{F} \to \mathbb{R}^d$ is piecewise–differentiable



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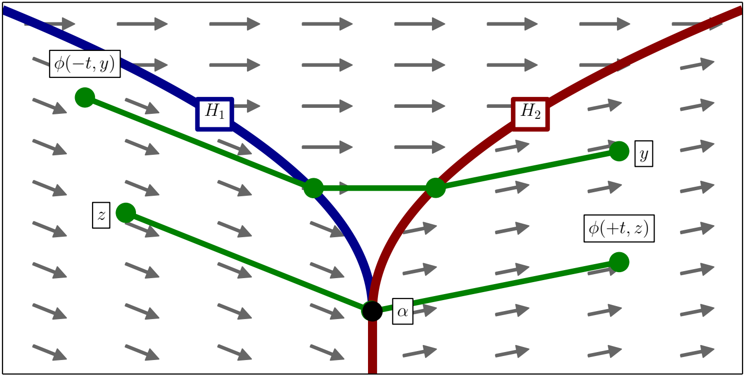

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ϕ possesses a nonclassical derivative $D\phi : T\mathcal{F} \to T\mathbb{R}^d$, i.e.

$$\forall (t, x) \in \mathcal{F} : \lim_{\delta \to 0} \frac{1}{\|\delta\|} \left\| \phi((t, x) + \delta) - (\phi(t, x) + D\phi(t, x; \delta)) \right\| = 0.$$

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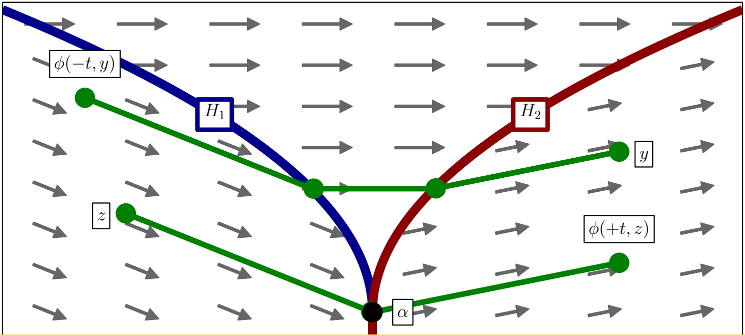

$D\phi$ is piecewise–affine; it satisfies chain rule, fundamental theorem of calculus, inverse & implicit function theorems

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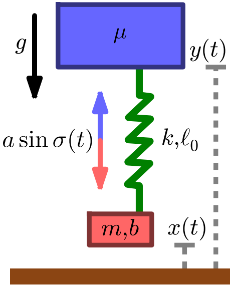
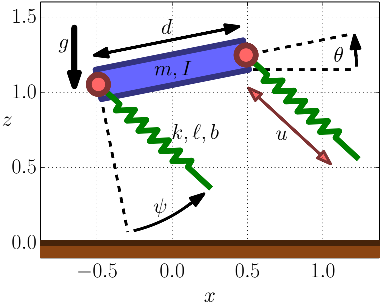
Implications for controlling dynamic and dexterous robots

1. Assess stability of nonsmooth Poincaré map $P : S \to \Sigma$ using nonclassical derivative $DP(\alpha)$ evaluated at fixed point $\alpha = P(\alpha)$.

2. Compute sensitivity of trajectory (i.e. *Lyapunov exponents*) w.r.t. state x and parameters ξ using nonclassical derivatives $D_x\phi$, $D_\xi\phi$.





3. Determine controllability by applying implicit function theorem to nonclassical derivative $D\phi$ of flow.

4. Perform scalable optimization of control inputs u using first– or second–order descent algorithms.

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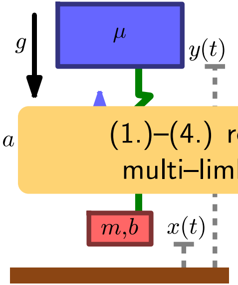

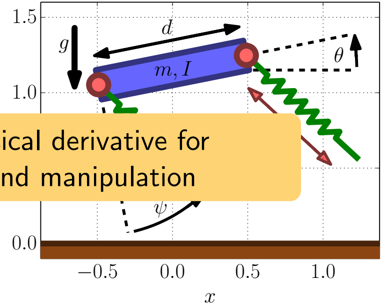

(1.)–(4.) require nonclassical derivative for multi–limb locomotion and manipulation

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Contribution from resolution of inconsistencies

Motivation: animals possess rich behavioral repertoire robots lack
Progress hampered by pathologies in parsimonious models.

1. Topological quotient removes discontinuities
   Enables convergent numerical simulation for legged locomotion.

2. Restricting impact restitution resolves inconsistencies
   Enables scalable nonsmooth optimization and control of locomotion.

Future directions: towards sensorimotor control theory
Synthesis and stabilization of rhythmic behaviors, aperiodic maneuvers.

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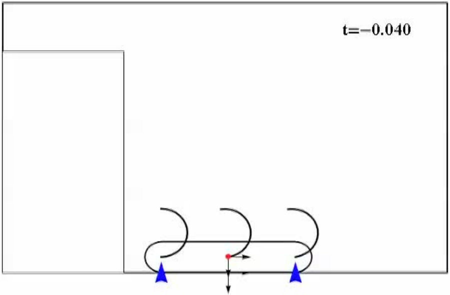
Dynamic & dexterous (self–)manipulation


Johnson & Koditschek ICRA 2013



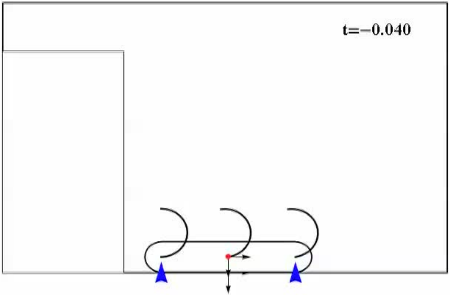
Dynamics with $n \in \mathbb{N}$ limbs, intrinsic coordinates $q \in Q$

continuous: $\ddot{q} = f(q, \dot{q}) + \lambda_J(q, \dot{q}) Da_J(q)$ discrete: $\dot{q}^+ = \Delta_J \dot{q}^-$

Johnson, Burden, Koditschek (*in prep*)
A Hybrid Systems Model for Simple Manipulation and Self–Manipulation Systems

Dynamic & dexterous (self–)manipulation

Johnson & Koditschek ICRA 2013



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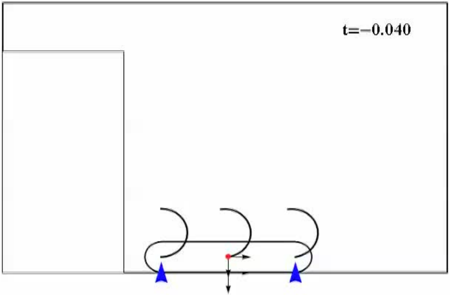

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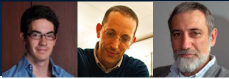
General framework accommodates variety of modeling assumptions.

Removing discontinuities and resolving inconsistencies enables new approaches to control, optimization, and planning.

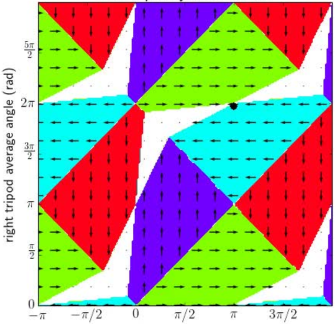
Johnson, Burden, Koditschek (*in prep*)
A Hybrid Systems Model for Simple Manipulation and Self–Manipulation Systems

Robost gaits exploit impact mechanics



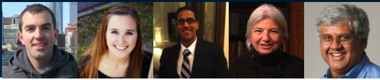
Exploit impacts to synchronize tripods

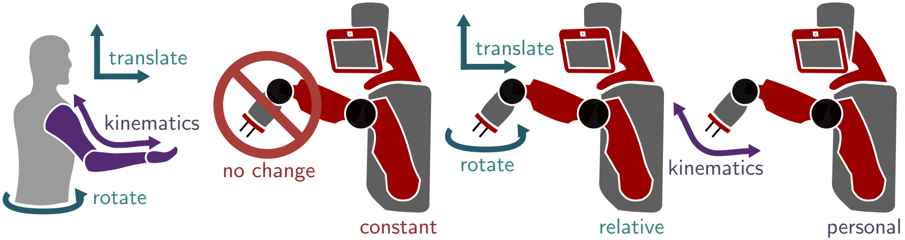
Introduce piecewise–constant feedback to enforce alternating–tripod gait.

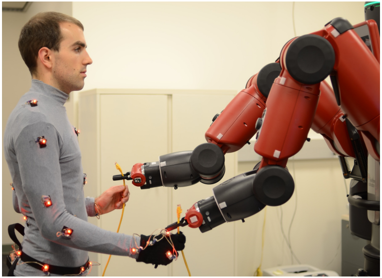


Kenneally, Burden, Revzen, Koditschek (*in prep*)

Collaborative manipulation







Kinematic model improves handoff

Dynamic model and intrinsic state space metric supports collaborative manipulation

Bestick, Burden, Willits, Naikal, Bajcsy, Sastry (*submitted to ICRA 2015*)

Discussion & Questions — Thanks for your time!

Discontinuities

Removed discontinuities from interaction betwen limbs and terrain.

Inconsistencies

Resolved inconsistencies from near–simultaneous limb touchdown.

Collaborators

- Shankar Sastry (UCB)
- Robert Full (UCB)
- Ruzena Bajcsy (UCB)
- Nikhil Naikal (UCB)
- Aaron Bestick (UCB)
- Giorgia Willits (UCB)
- Dan Koditschek (UPenn)
- Aaron Johnson (UPenn)
- Gavin Kenneally (UPenn)
- Shai Revzen (UMich)

Funding

- NSF (Award #1427260)
- ONR MURI (ONR N000141310341)
- ARL MAST CTA (W911NF-08-2-0004)