Metrization, Simulation, and First–Order Approximation for Networked CPS

Sam Burden

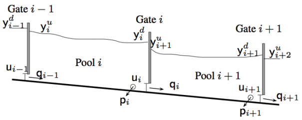
currently:
Postdoctoral Researcher
EECS Department
University of California, Berkeley

from Fall 2015:
Assistant Professor
EE Department
University of Washington, Seattle

Dynamics of CPS are nonclassical

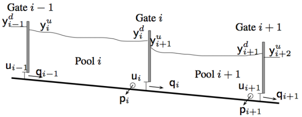
Dynamics of CPS are nonclassical

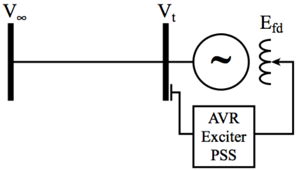
Amin, Litrico, Sastry, Bayen 2013



Dynamics of CPS are nonclassical

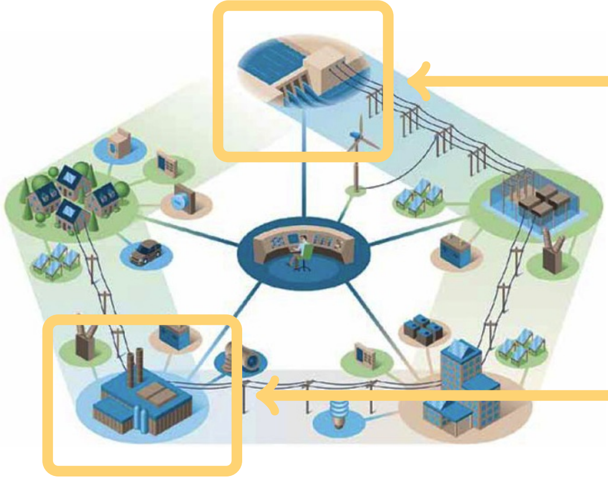
Amin, Litrico, Sastry, Bayen 2013



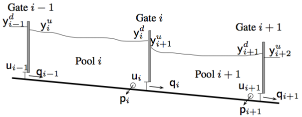


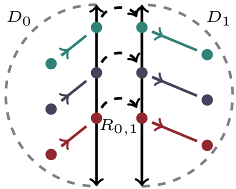
Hiskens & Reddy 2007

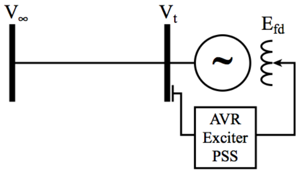
Dynamics of CPS are nonclassical



Amin, Litrico, Sastry, Bayen 2013

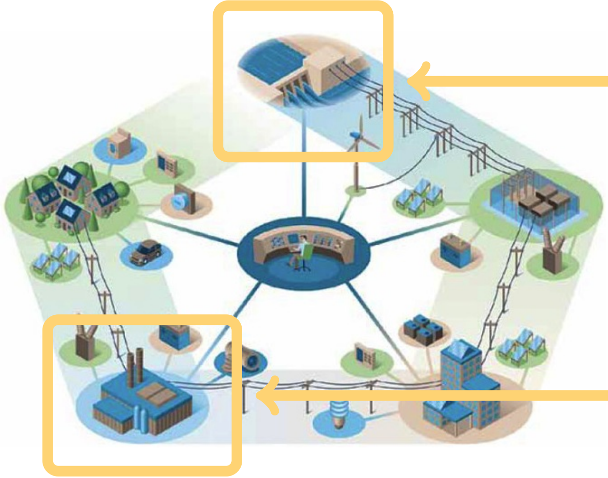




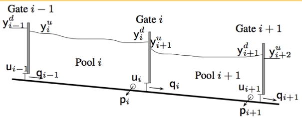


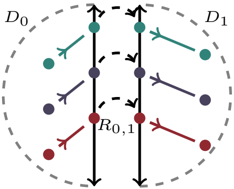
Hiskens & Reddy 2007

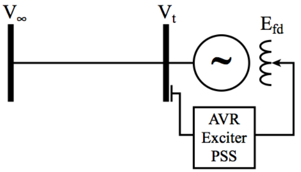
Dynamics of CPS are nonclassical



Amin, Litrico, Sastry, Bayen 2013





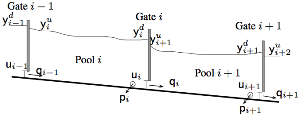


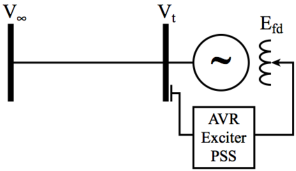
Hiskens & Reddy 2007

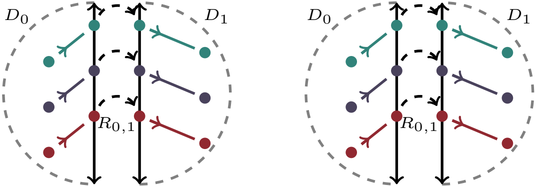
Isolated transitions can be “smoothed”

(arXiv:1308.4158)

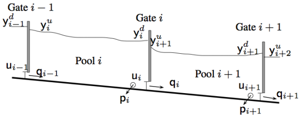
Networked CPS undergo interdependent transitions

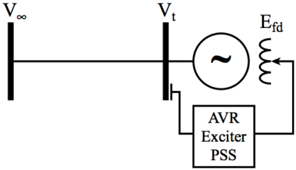


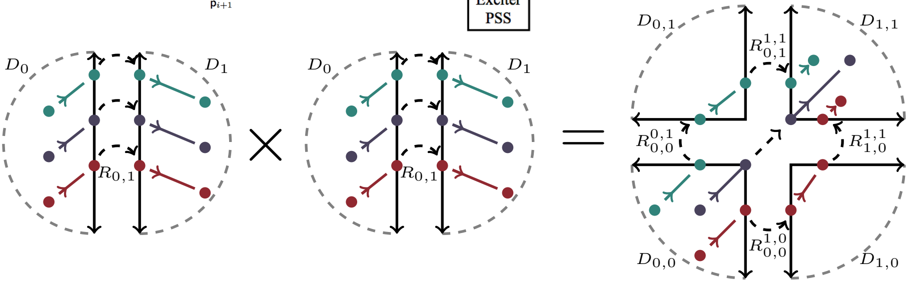




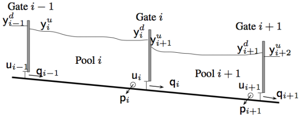
Networked CPS undergo interdependent transitions

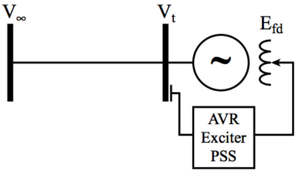


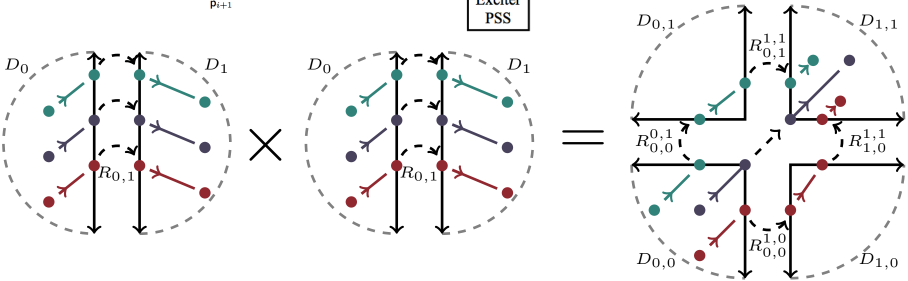




Networked CPS undergo interdependent transitions





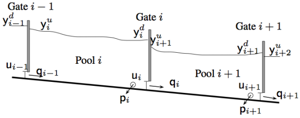


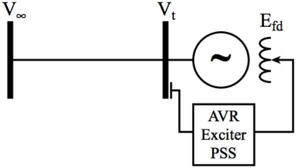
Implications of interdependent transitions

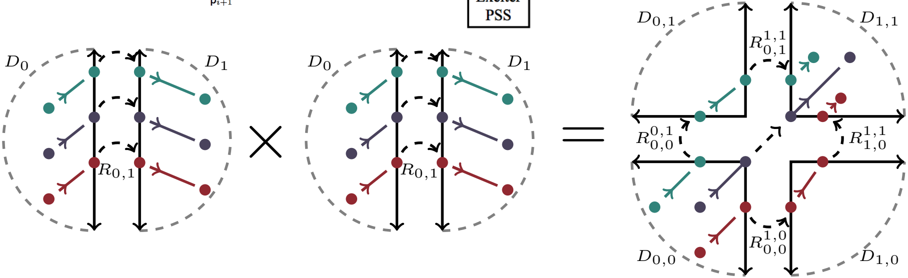
— Combinatorial increase in complexity

— Intrinsically nonsmooth transitions

Networked CPS undergo interdependent transitions







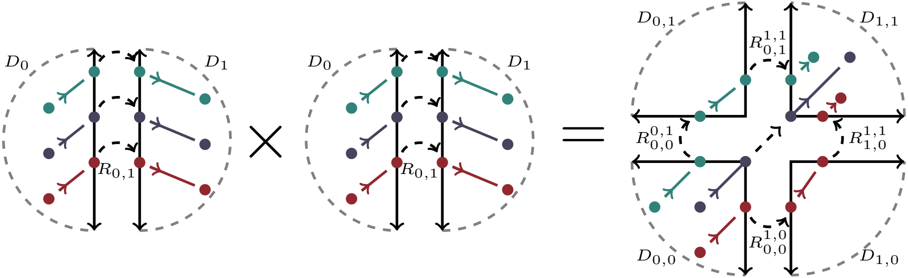
Implications of interdependent transitions

— Combinatorial increase in complexity

— Intrinsically nonsmooth transitions

New challenges require new tools

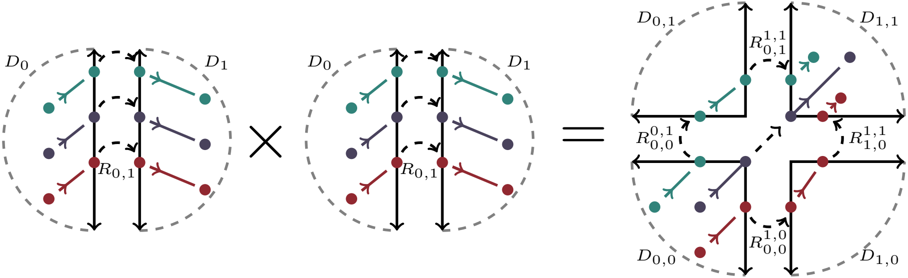
Metrization, Simulation, and First–Order Approximation



1. Metrization & Simulation

2. First–Order Approximation

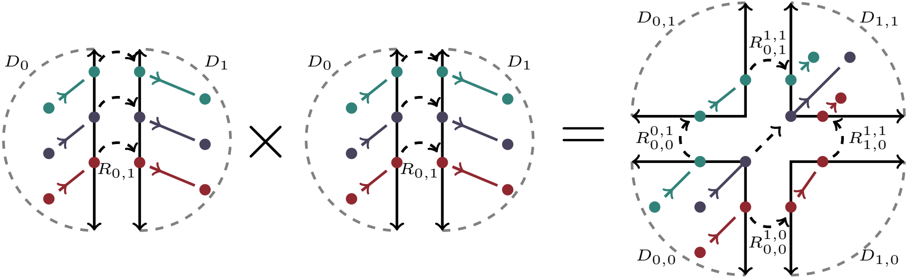
Metrization & Simulation for networked CPS



1. Metrization & Simulation

2. First–Order Approximation

Metrization & Simulation for networked CPS



1. Metrization & Simulation

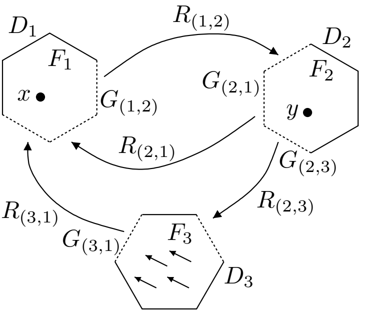
Using intrinsic state space metric, simulations converge uniformly and at a linear rate.

2. First–Order Approximation

Distance metric and simulation algorithm

Hybrid control systems comprised of distinct operating "modes"

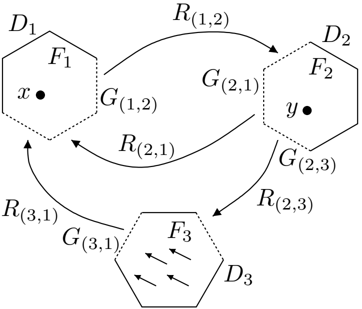
- State of supervisory control ("on" or "off")
- Physical/dynamical regime (switches, shocks, & saturation)

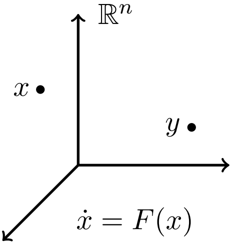


Distance metric and simulation algorithm

Hybrid control systems comprised of distinct operating "modes"

- State of supervisory control ("on" or "off")
- Physical/dynamical regime (switches, shocks, & saturation)





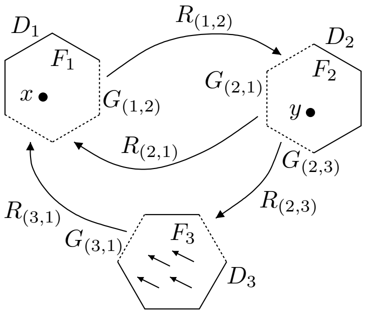
Classical ODE system

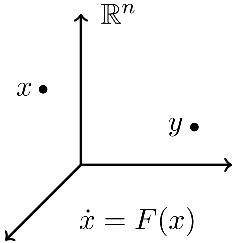
- distance: $d(x, y) = \|x - y\|$
- simulation: $x_{k+1} = x_k + hF(x_k)$

Distance metric and simulation algorithm

Hybrid control systems comprised of distinct operating "modes"

- State of supervisory control ("on" or "off")
- Physical/dynamical regime (switches, shocks, & saturation)





Hybrid dynamical system

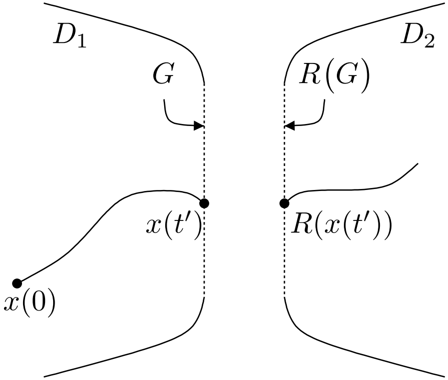
- distance: $d(x, y) = \infty$
- simulation: $x_k + hF(x_k) \notin D$

Classical ODE system

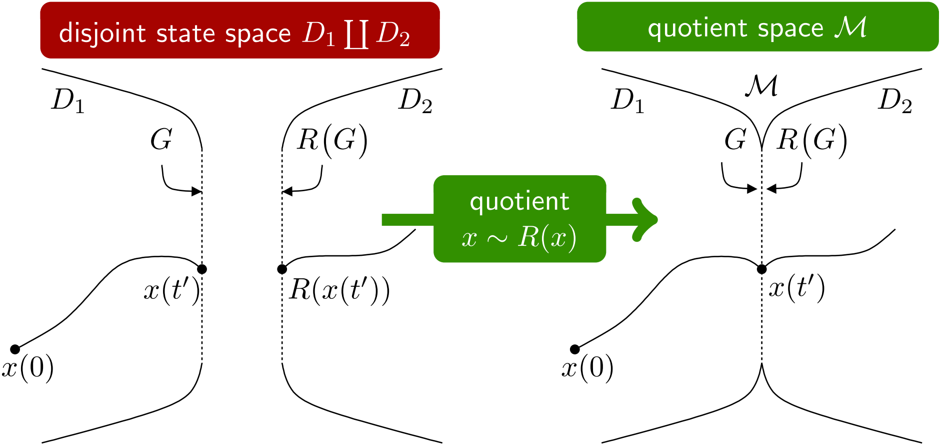
- distance: $d(x, y) = \|x - y\|$
- simulation: $x_{k+1} = x_k + hF(x_k)$

Remove discontinuities via topological quotient

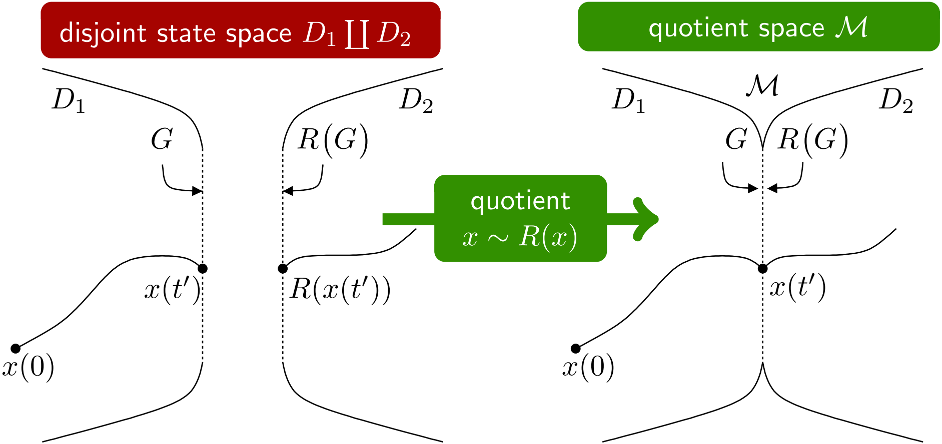
disjoint state space $D_1 \coprod D_2$



Remove discontinuities via topological quotient



Remove discontinuities via topological quotient

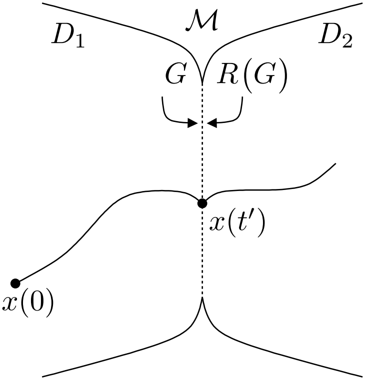


Theorem (arXiv:1302.4402)

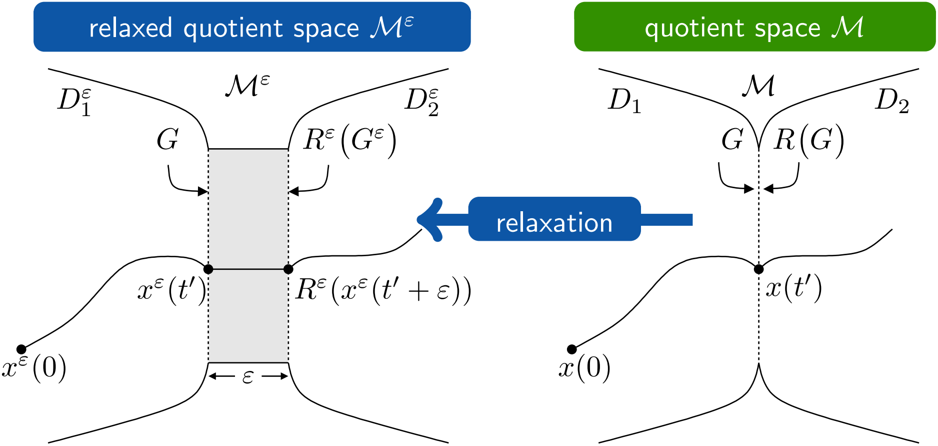
$\mathcal{M}$ is metrizable.

Relax quotient space at discrete transitions

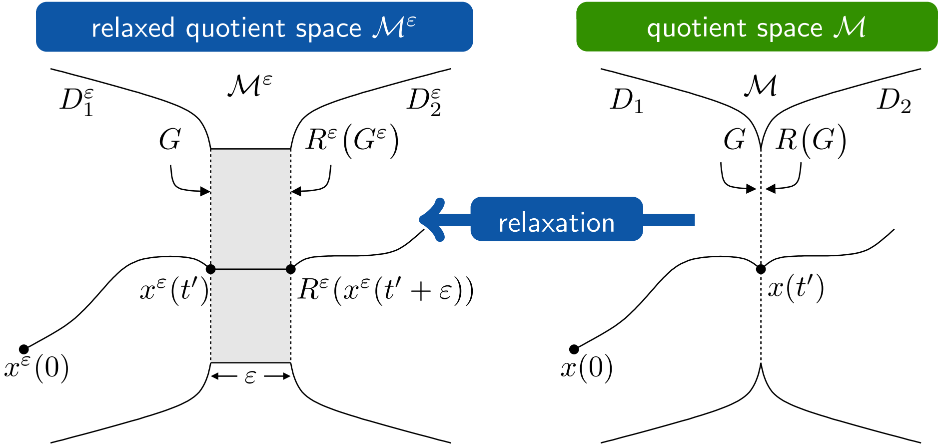
quotient space $\mathcal{M}$



Relax quotient space at discrete transitions



Relax quotient space at discrete transitions



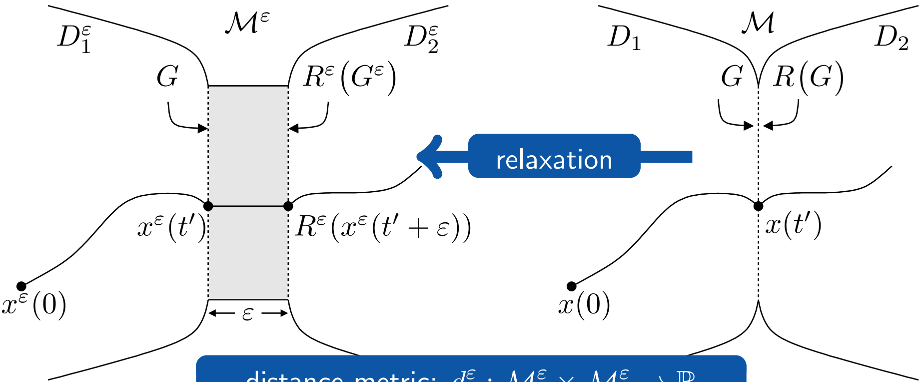
Theorem (arXiv:1302.4402)

$\mathcal{M}^\varepsilon$ is metrizable.

Relax quotient space at discrete transitions

relaxed quotient space $\mathcal{M}^\varepsilon$

quotient space $\mathcal{M}$



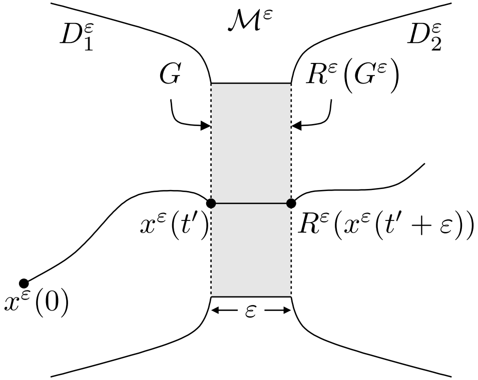
distance metric: $d^\varepsilon : \mathcal{M}^\varepsilon \times \mathcal{M}^\varepsilon \to \mathbb{R}$

Theorem (arXiv:1302.4402)

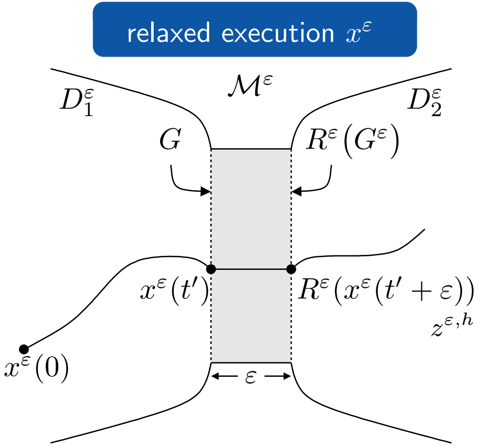
$\mathcal{M}^\varepsilon$ is metrizable.

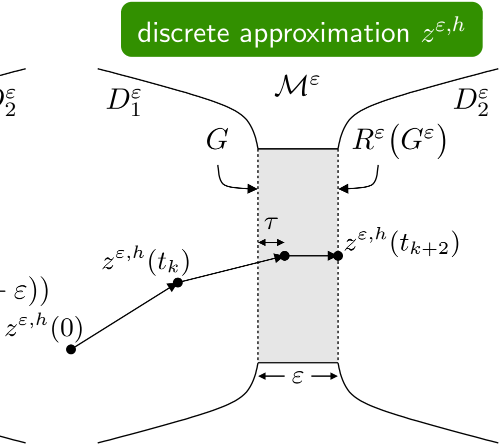
Numerical simulation on relaxed quotient space

relaxed execution x^ε



Numerical simulation on relaxed quotient space

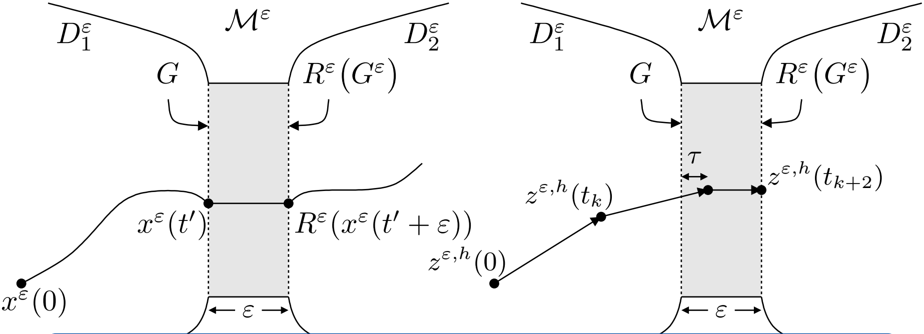




Numerical simulation on relaxed quotient space

relaxed execution x^ε

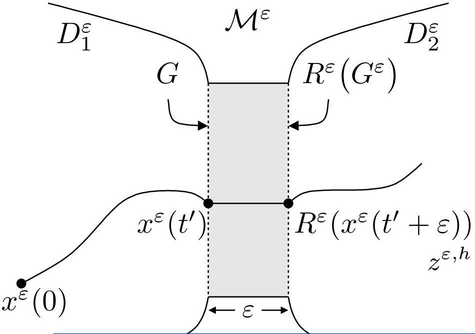
discrete approximation $z^{\varepsilon,h}$



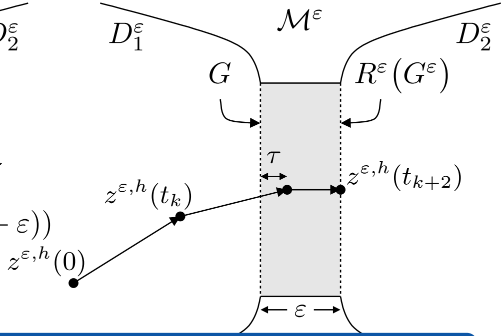
trajectory metric: $\rho^\varepsilon(x, z^{\varepsilon,h}) = \sup\left\{ d^\varepsilon(x^\varepsilon(s), z^{\varepsilon,h}(s)) : s \in [0,t] \right\}$

Numerical simulation on relaxed quotient space

relaxed execution x^ε



discrete approximation $z^{\varepsilon,h}$



trajectory metric: $\rho^\varepsilon(x, z^{\varepsilon,h}) = \sup\left\{d^\varepsilon(x^\varepsilon(s), z^{\varepsilon,h}(s)) : s \in [0,t]\right\}$

Theorem (arXiv:1302.4402)

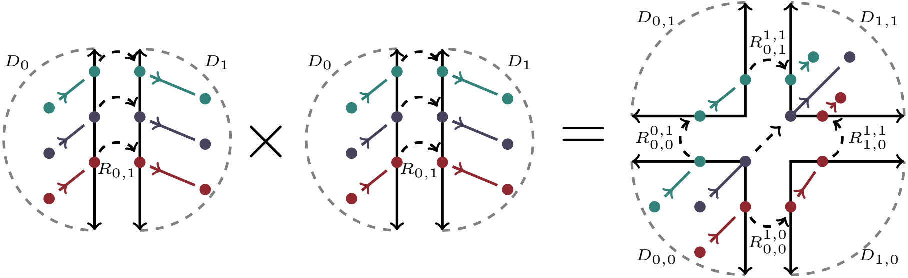
If x is orbitally stable then $\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) \in O(\varepsilon) + O(h)$.

Implications for networked CPS

Intrinsic state space metric and convergent numerical simulation

— Quantification of performance degradation through discrete transitions
— Reliable simulation for model–based design and predictive control

Contribution from metrization & simulation



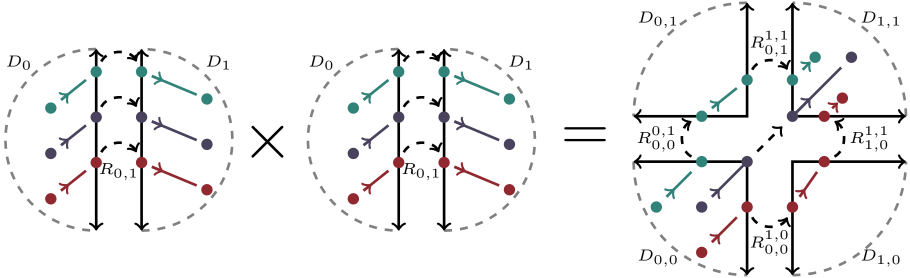
1. Metrization & Simulation

Using intrinsic metric space $(M^\varepsilon, d^\varepsilon)$, simulations converge:

$$\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) = O(\varepsilon) + O(h)$$

2. First–Order Approximation

First–Order Approximation for networked CPS



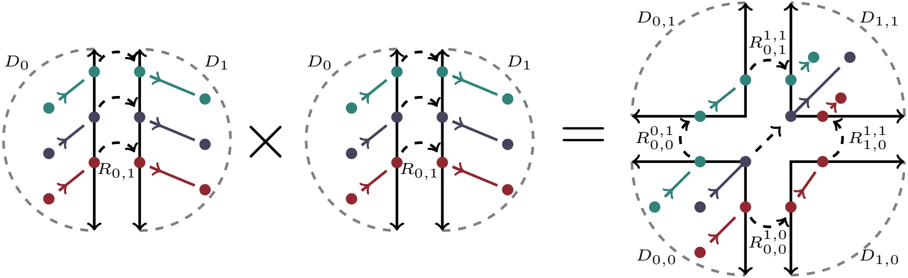
1. Metrization & Simulation

Using intrinsic metric space $(M^\varepsilon, d^\varepsilon)$, simulations converge:

$$\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) = O(\varepsilon) + O(h)$$

2. First–Order Approximation

First–Order Approximation for networked CPS



1. Metrization & Simulation

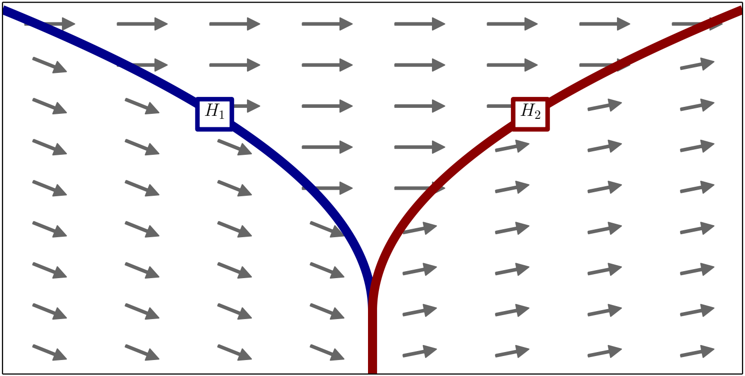
Using intrinsic metric space $(M^\varepsilon, d^\varepsilon)$, simulations converge:

$$\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) = O(\varepsilon) + O(h)$$

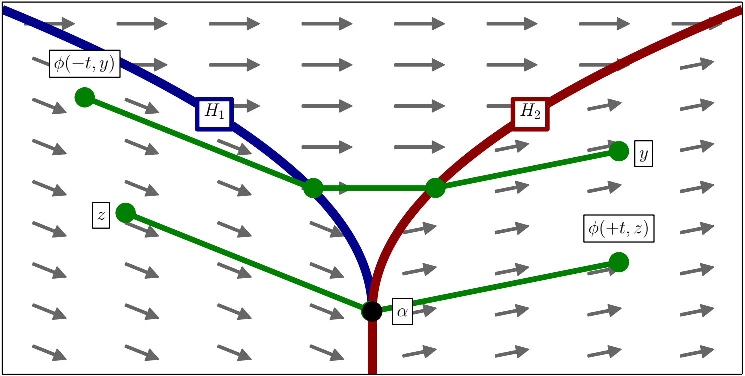
2. First–Order Approximation

Nonsmooth flow of networked CPS is piecewise–differentiable; can approximate it using a nonclassical "derivative".

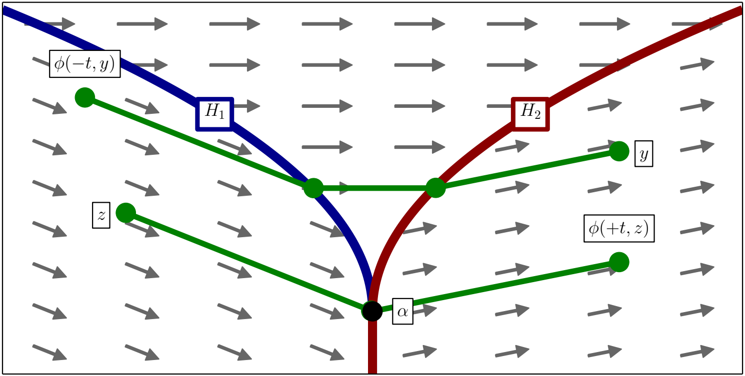
Discrete transitions lead to discontinuous dynamics



Discrete transitions lead to discontinuous dynamics



Discrete transitions lead to discontinuous dynamics

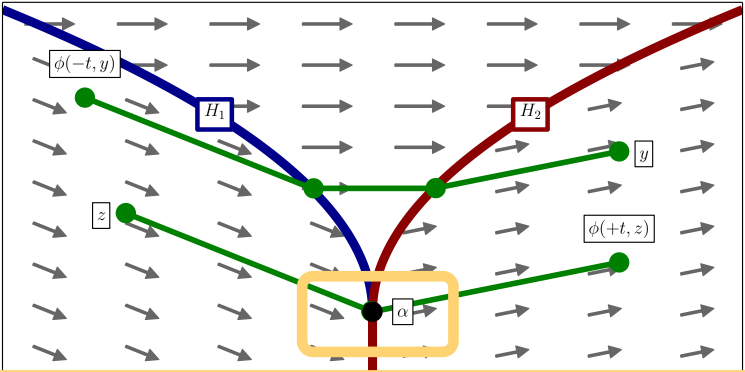


Theorem (arXiv:1407.1775)

Discontinuous vector field $\dot{x} = F(x)$ *yields nonsmooth flow* $\phi : \mathcal{F} \to \mathbb{R}$*:*

$$\forall (t, x) \in \mathcal{F} \subset \mathbb{R} \times \mathbb{R}^d : \phi(t, x) = x + \int_0^t F(\phi(s, x))\, ds.$$

Discrete transitions lead to discontinuous dynamics



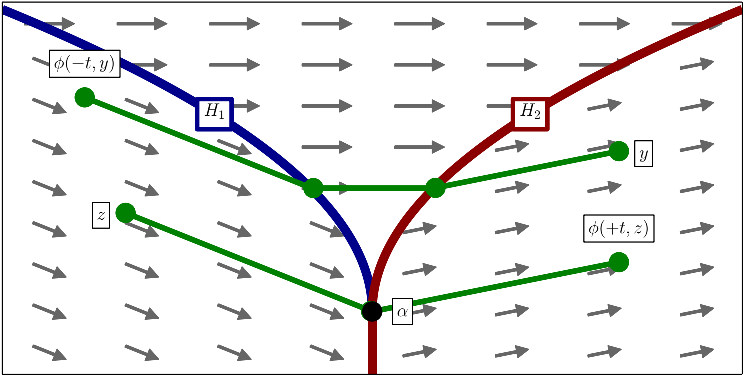
ϕ is nonsmooth since $D_t\phi$ is undefined e.g. at $\alpha \in H_1 \cap H_2$

Theorem (arXiv:1407.1775)

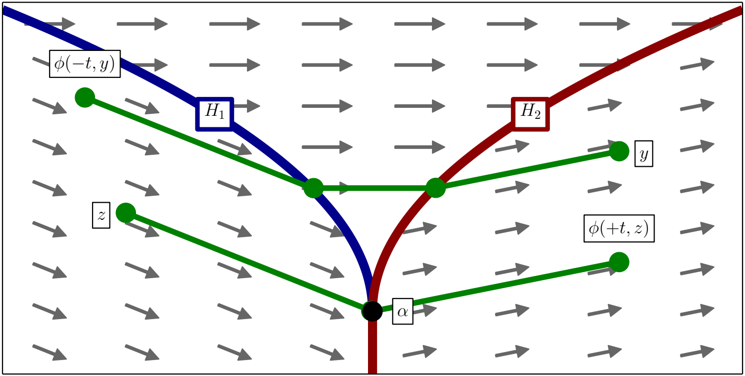
Discontinuous vector field $\dot{x} = F(x)$ *yields nonsmooth flow* $\phi : \mathcal{F} \to \mathbb{R}$:

$$\forall (t, x) \in \mathcal{F} \subset \mathbb{R} \times \mathbb{R}^d : \phi(t, x) = x + \int_0^t F\left(\phi(s, x)\right) ds.$$

Nonsmooth flow $\phi : \mathcal{F} \to \mathbb{R}^d$ is piecewise–differentiable



Nonsmooth flow $\phi : \mathcal{F} \to \mathbb{R}^d$ is piecewise–differentiable

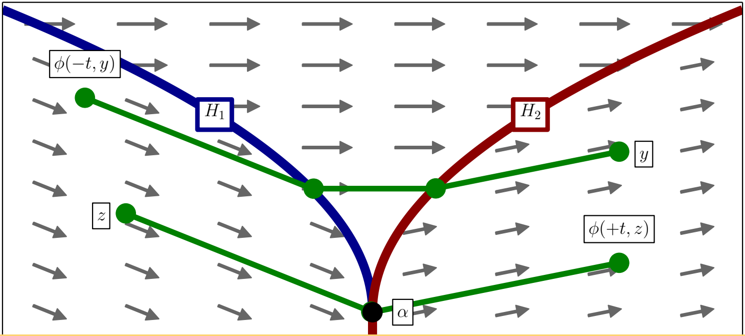


Theorem (arXiv:1407.1775)

ϕ possesses a nonclassical derivative $D\phi : T\mathcal{F} \to T\mathbb{R}^d$, i.e.

$$\forall (t, x) \in \mathcal{F} : \lim_{\delta \to 0} \frac{1}{\|\delta\|} \|\phi((t, x) + \delta) - (\phi(t, x) + D\phi(t, x; \delta))\| = 0.$$

Nonsmooth flow $\phi : \mathcal{F} \to \mathbb{R}^d$ is piecewise–differentiable



$D\phi$ is piecewise–affine; it satisfies chain rule, fundamental theorem of calculus, inverse & implicit function theorems

Theorem (arXiv:1407.1775)

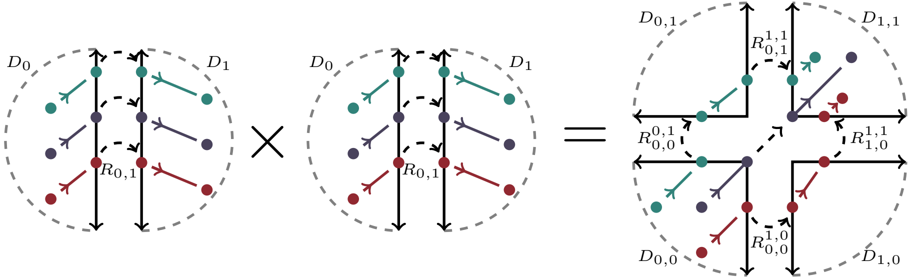
ϕ possesses a nonclassical derivative $D\phi : T\mathcal{F} \to T\mathbb{R}^d$, i.e.

$$\forall (t, x) \in \mathcal{F} : \lim_{\delta \to 0} \frac{1}{\|\delta\|} \|\phi((t, x) + \delta) - (\phi(t, x) + D\phi(t, x; \delta))\| = 0.$$

Implications for networked CPS

1. Assess stability of nonsmooth Poincaré map $P : S \to \Sigma$ using nonclassical derivative $DP(\alpha)$ evaluated at fixed point $\alpha = P(\alpha)$.

2. Compute sensitivity of trajectory (i.e. *Lyapunov exponents*) w.r.t. state x and parameters ξ using nonclassical derivatives $D_x\phi$, $D_\xi\phi$.



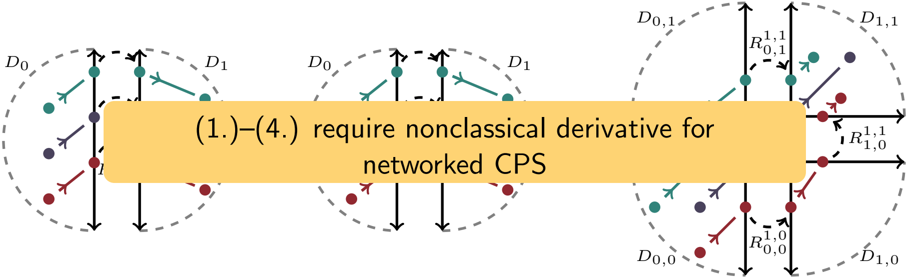
3. Determine controllability by applying implicit function theorem to nonclassical derivative $D\phi$ of flow.

4. Perform scalable optimization of control inputs u using first– or second–order descent algorithms.

Implications for networked CPS

1. Assess stability of nonsmooth Poincaré map $P : S \to \Sigma$ using nonclassical derivative $DP(\alpha)$ evaluated at fixed point $\alpha = P(\alpha)$.

2. Compute sensitivity of trajectory (i.e. *Lyapunov exponents*) w.r.t. state x and parameters ξ using nonclassical derivatives $D_x\phi$, $D_\xi\phi$.

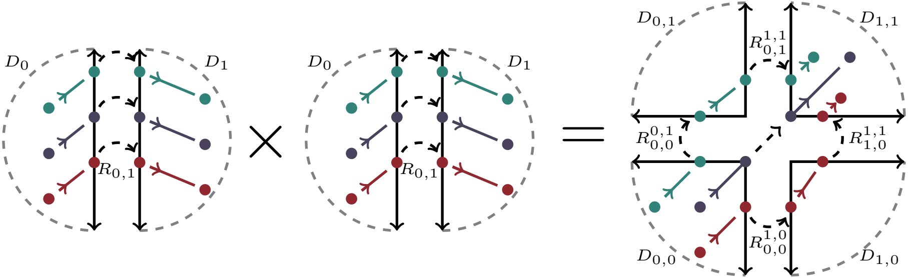


(1.)–(4.) require nonclassical derivative for networked CPS

3. Determine controllability by applying implicit function theorem to nonclassical derivative $D\phi$ of flow.

4. Perform scalable optimization of control inputs u using first– or second–order descent algorithms.

Contribution from first–order approximation



1. Metrization & Simulation

Using intrinsic metric space $(M^\varepsilon, d^\varepsilon)$, simulations converge:

$$\rho^\varepsilon(x^\varepsilon, z^{\varepsilon,h}) = O(\varepsilon) + O(h)$$

2. First–Order Approximation

Nonsmooth flow $\phi : \mathcal{F} \to D$ is piecewise–differentiable:

$$\phi(t+u, x+v) = \phi(t,x) + D\phi(t,x;u,v) + O(|u|^2 + \|v\|^2)$$

Discussion & Questions — Thanks for your time!

1. Metrization & Simulation

Intrinsic state space metric and convergent simulation algorithm. (arXiv:1302.4402)

2. First–Order Approximation

Nonsmooth dynamics of networked CPS are piecewise–differentiable. (arXiv:1407.1775)

Collaborators

- Shankar Sastry (UCB)
- Ruzena Bajcsy (UCB)
- Dan Koditschek (UPenn)
- Shai Revzen (UMich)
- Humberto Gonzalez (WUSTL)
- Ram Vasudevan (UMich)

Sponsors

- NSF FORCES (CNS #1238959)
- ARL MAST CTA (W911NF-08-2-0004)