Parsimonious predictive models for legged locomotion

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currently:
Postdoctoral Scholar
EECS Department
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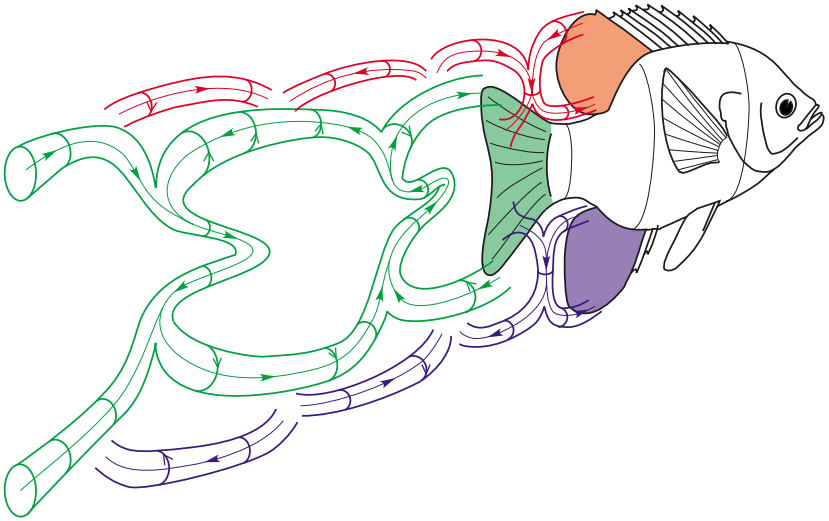
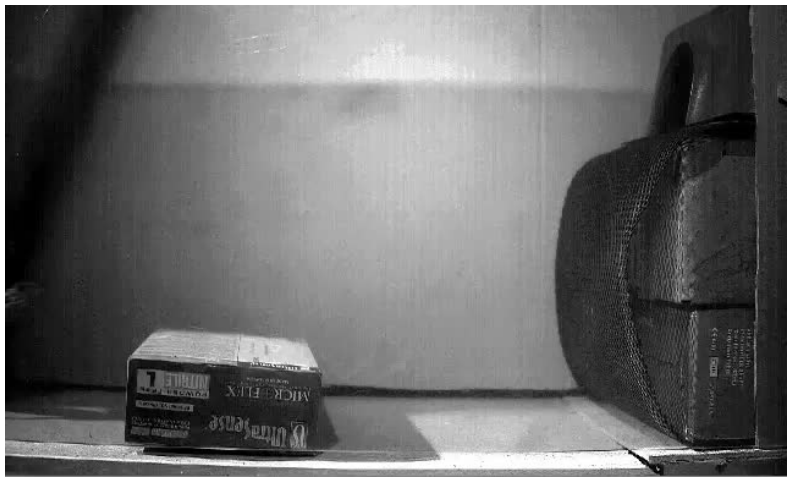
Assistant Professor
EE Department
University of Washington, Seattle

Legged locomotion involves *intermittent* interaction

hydro–dynamics



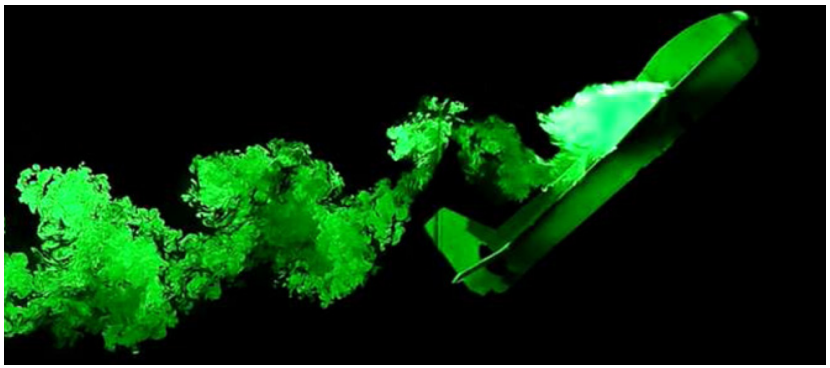
Tytell, Standen, Lauden JEB 2008

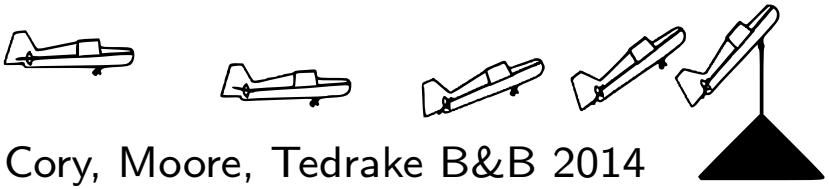


Libby, Moore, Chang–Siu, Li, Cohen, Jusufi, Full Nature 2012

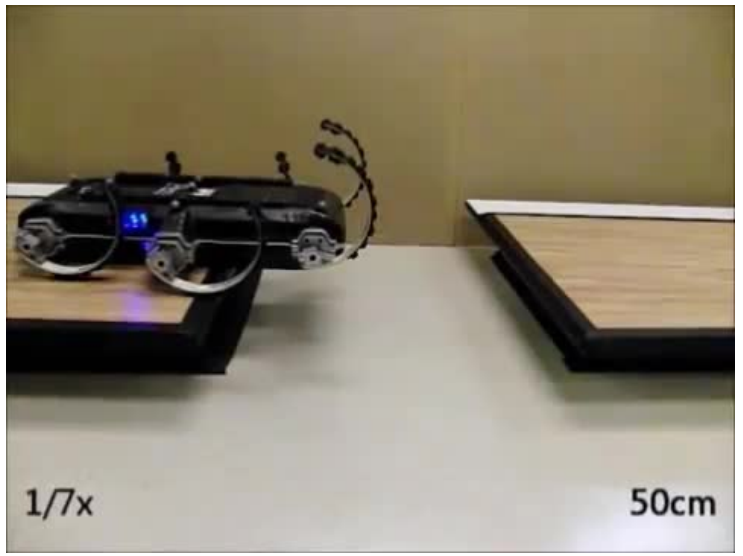
terra–dynamics

aero–dynamics



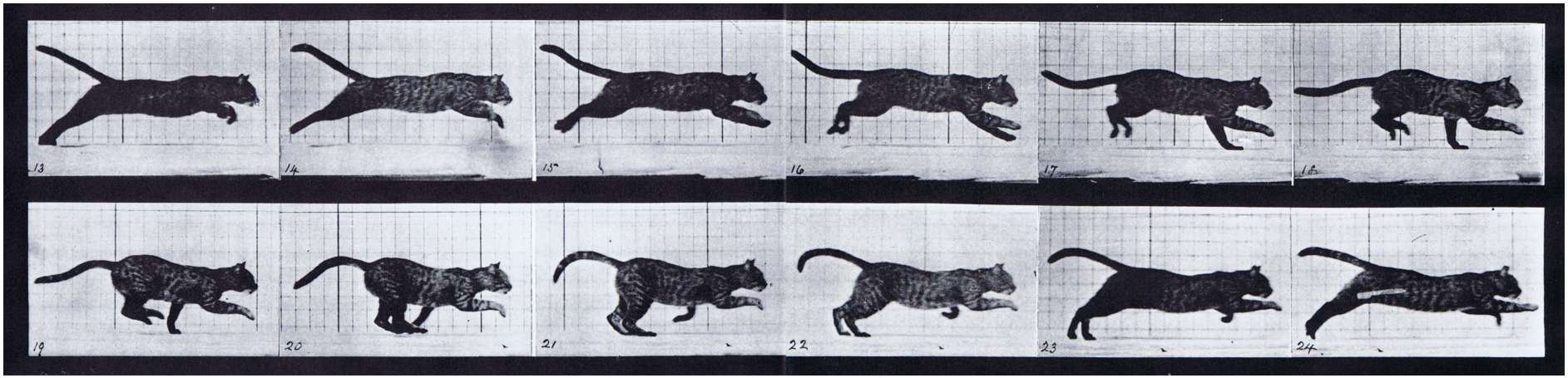


Cory, Moore, Tedrake B&B 2014



Johnson & Koditschek ICRA 2013

Parsimonious models are piecewise–defined



Muybridge 1957

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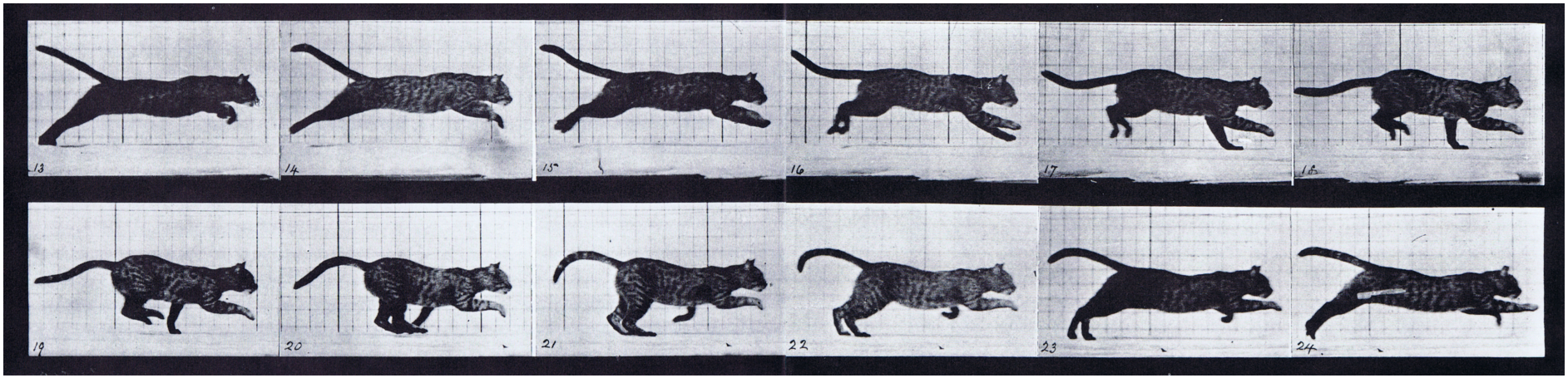


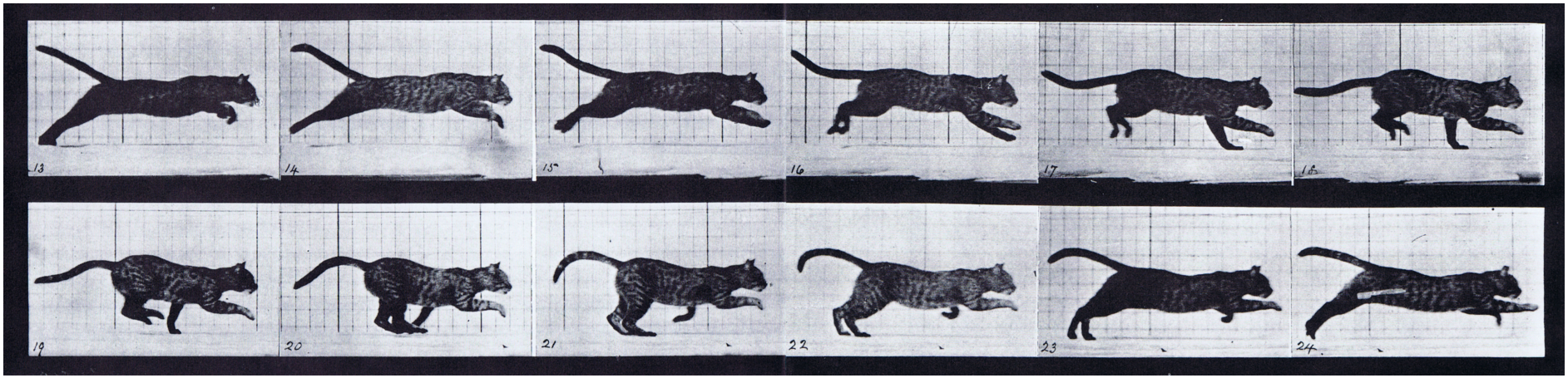
Muybridge 1957

Dynamics with $n \in \mathbb{N}$ limbs, intrinsic coordinates $q \in Q$

- Each subset of contact limbs $J \subset \{1, \ldots, n\}$ determine continuous dynamics $\ddot{q} = f(q, \dot{q}) + \lambda_J(q, \dot{q}) D a_J(q)$ subject to constraints $a_J(q) \equiv 0$.
- At impact into mode J, velocities update discontinuously: $\dot{q}^+ = \Delta_J \dot{q}^-$.

Johnson, Burden, Koditschek (arXiv:1502.01538)
A hybrid systems model for simple manipulation and self–manipulation systems

Parsimonious models are piecewise–defined



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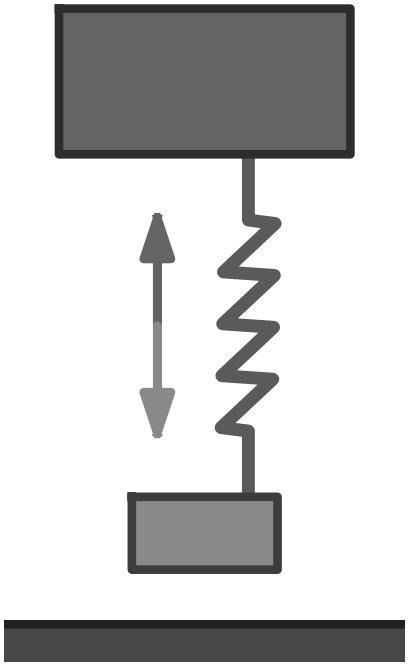
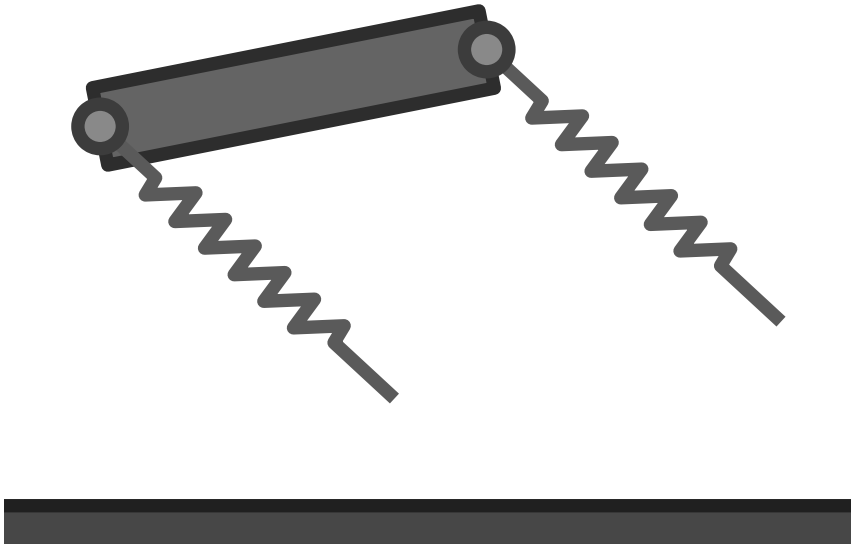
Yields a piecewise–defined ("hybrid") model for legged locomotion.

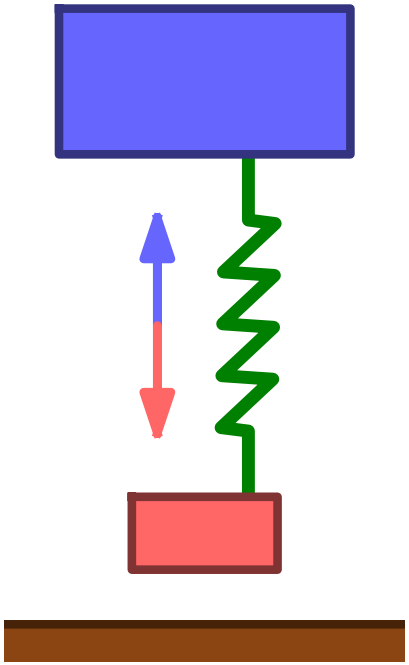
Johnson, Burden, Koditschek (arXiv:1502.01538)
A hybrid systems model for simple manipulation and self–manipulation systems

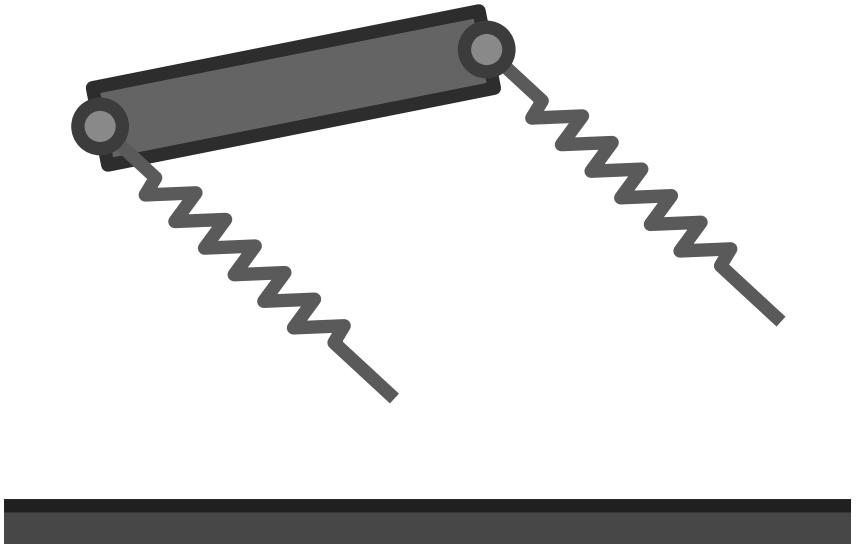
Pathologies in piecewise–defined models





Pathologies in piecewise–defined models

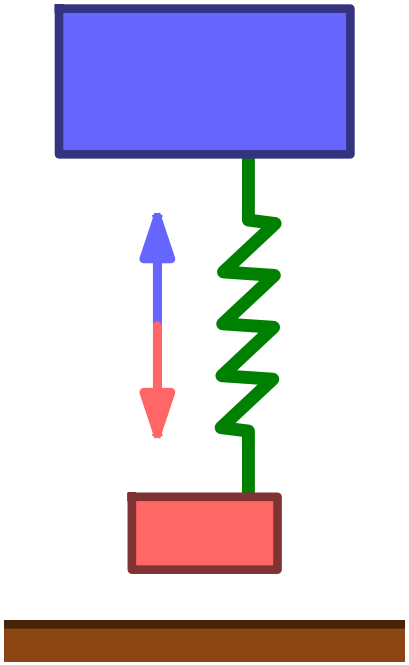


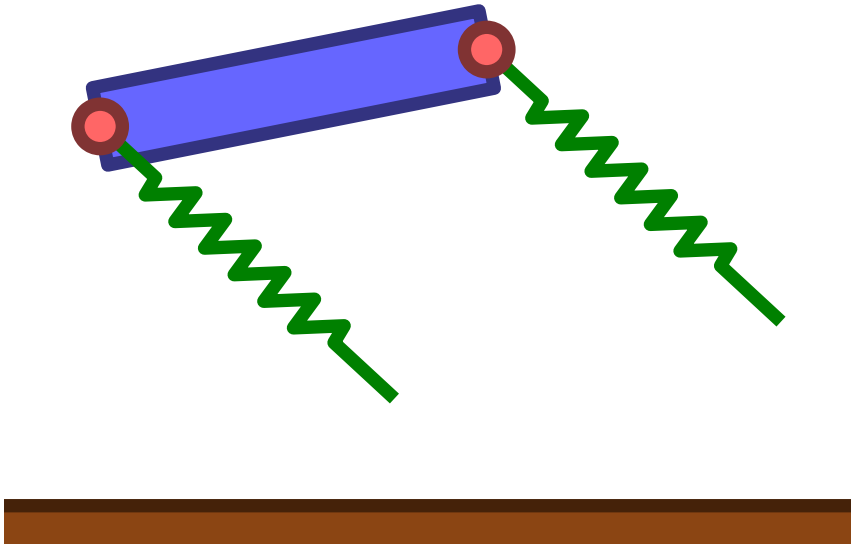


1. Discontinuities

equations–of–motion and states change abruptly at impact

Pathologies in piecewise–defined models





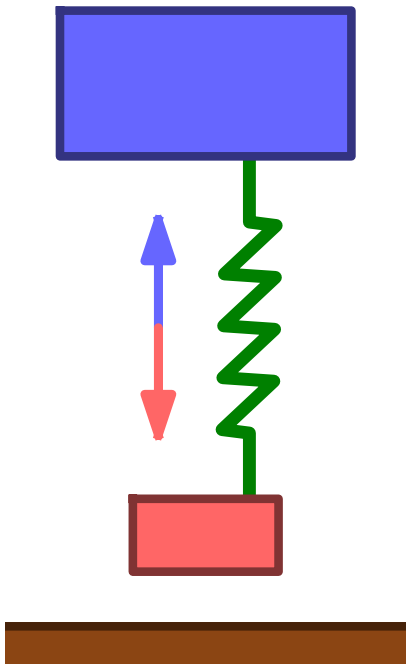
1. Discontinuities

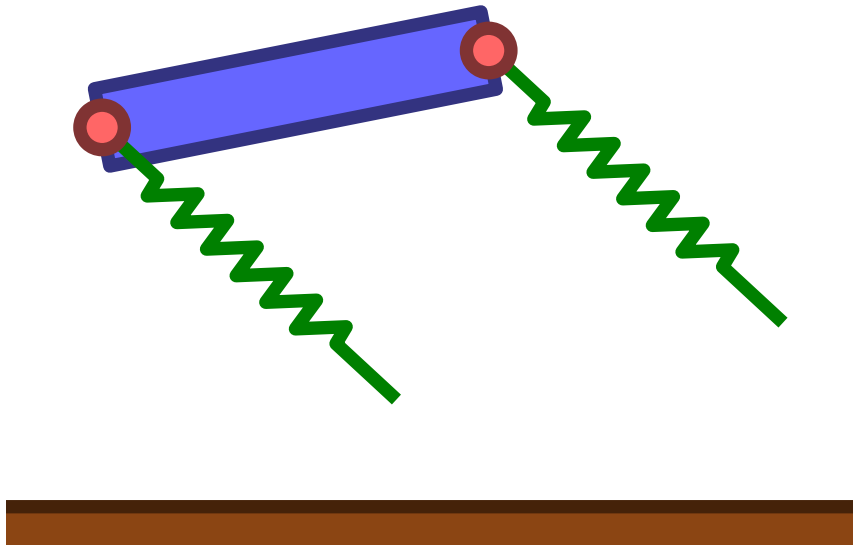
equations–of–motion and states change abruptly at impact

2. Inconsistencies

restitution laws lead to nondeterminism at impact

Pathologies in piecewise–defined models





1. Discontinuities

equations–of–motion and states change abruptly at impact

2. Inconsistencies

restitution laws lead to nondeterminism at impact

Prevents application of classical tools for prediction or design.

Today's talk

Motivation: legged locomotion involves *intermittent* interaction

Predictions limited by pathologies in parsimonious models.

1. Mathematical "glue" removes discontinuities

   Yields reliable simulation algorithm and novel route to model reduction.

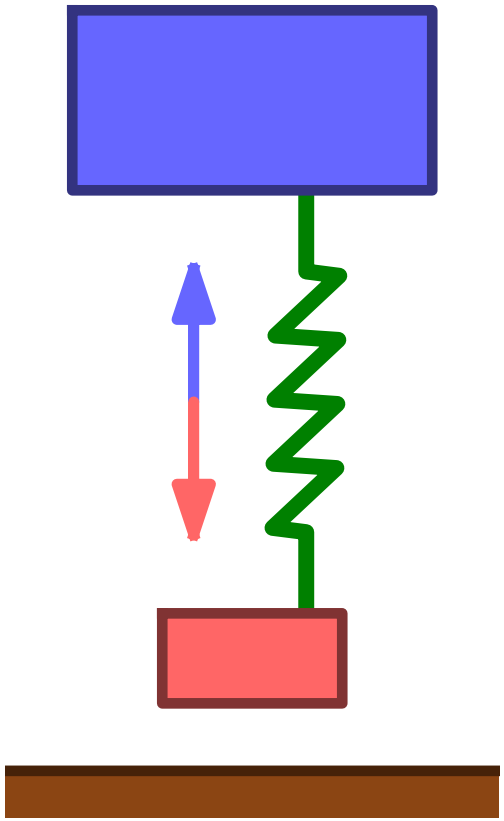
2. Restricting impact restitution resolves inconsistencies

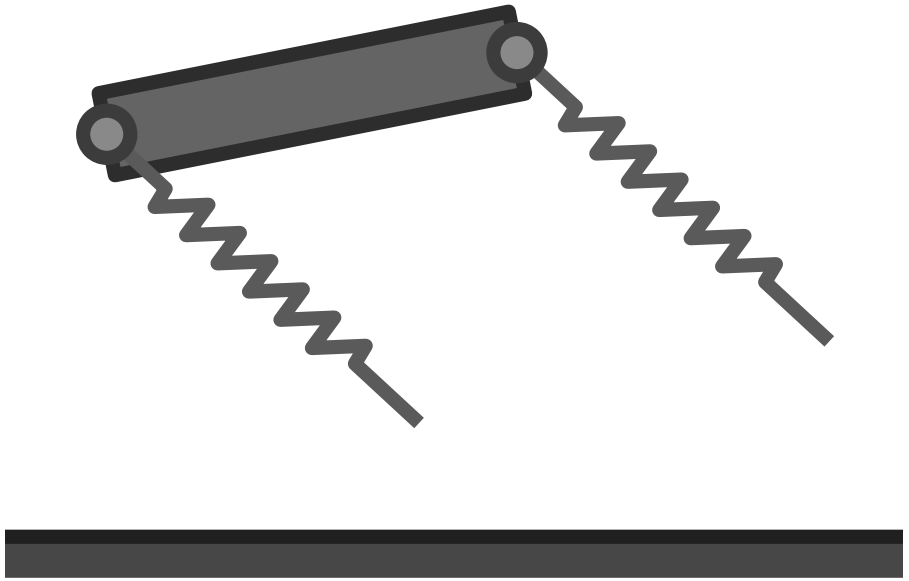
   Yields scalable optimization algorithm and novel route to stabilization.

Future directions: predictions for robotics & biology

Effect of parameters and perturbations on gaits and maneuvers.

Parsimonious predictive models for legged locomotion

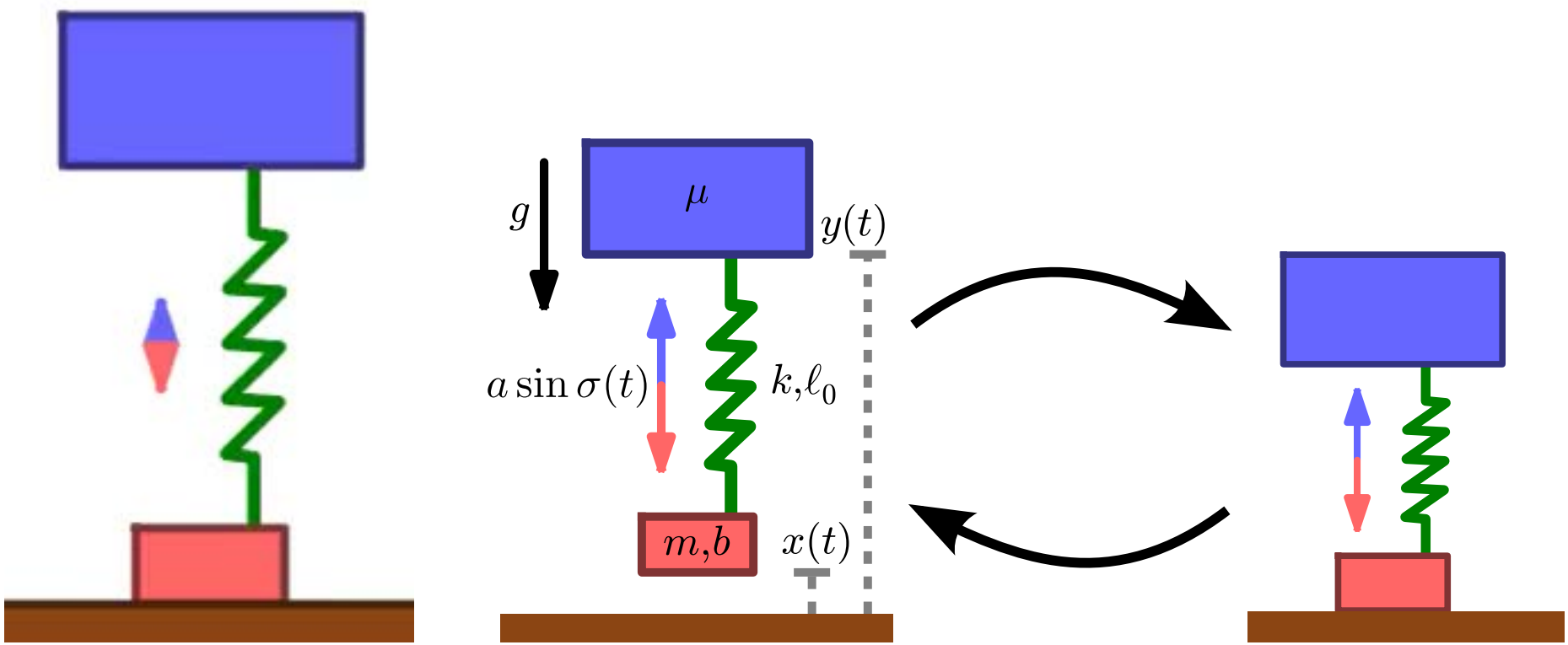




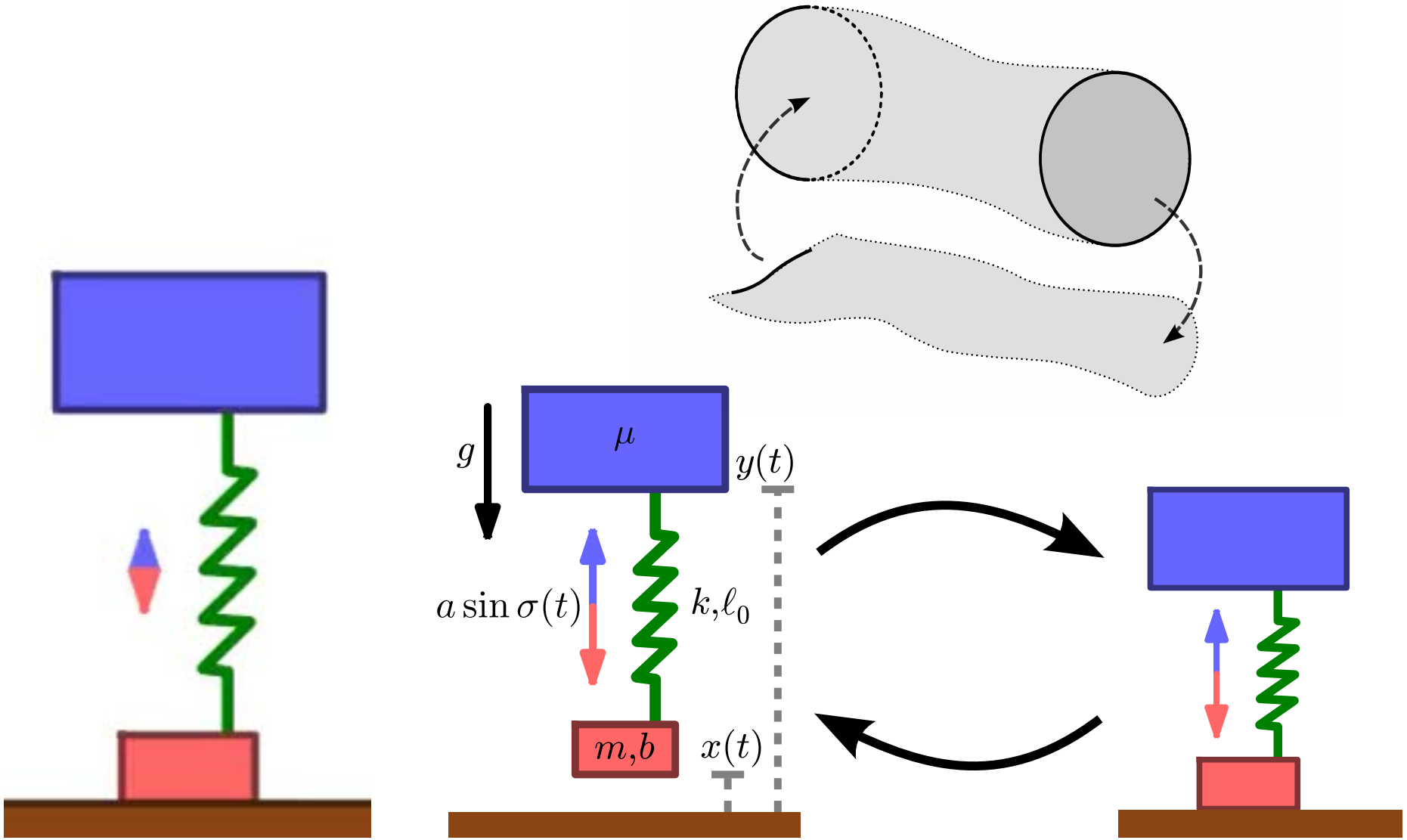
1. Remove discontinuities

2. Resolve inconsistencies

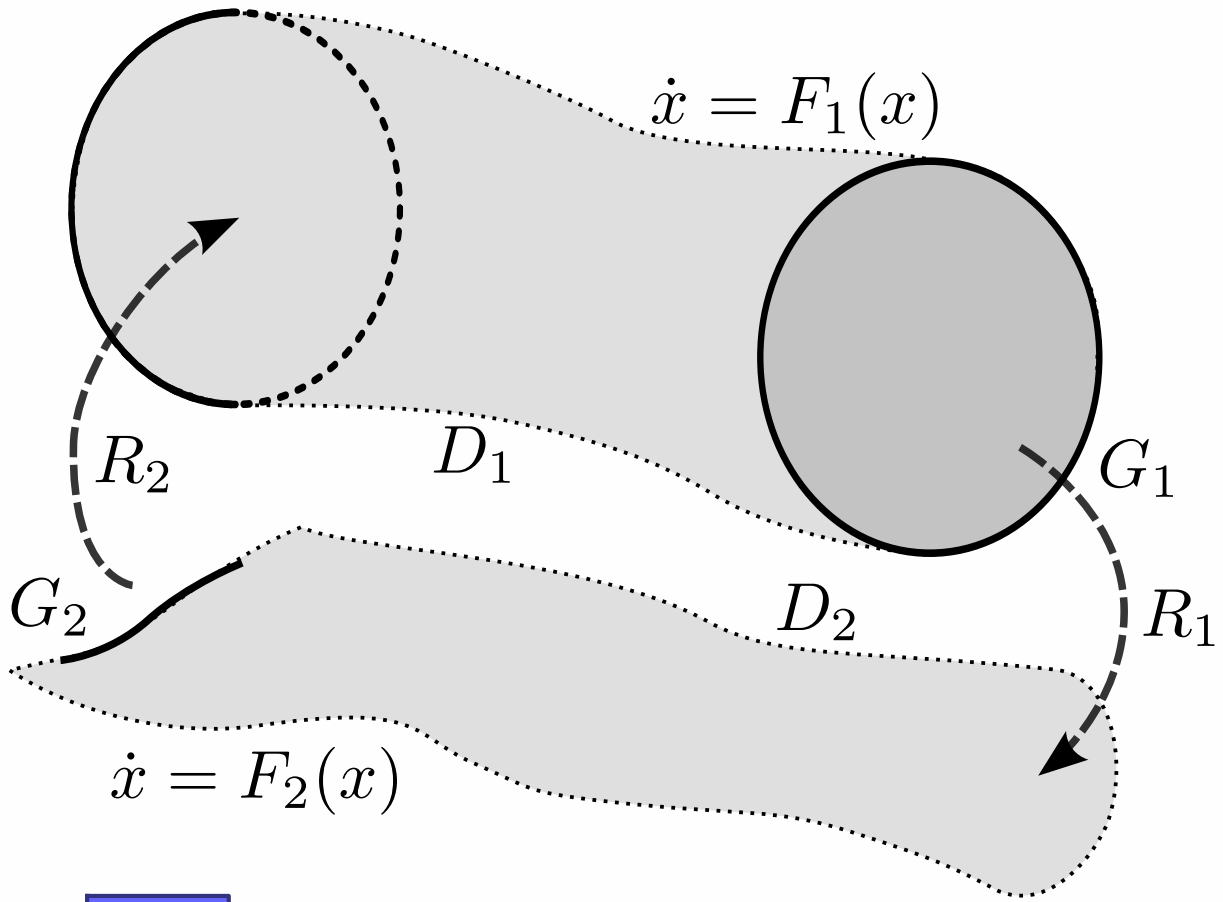
Discontinuities in vertical hopping

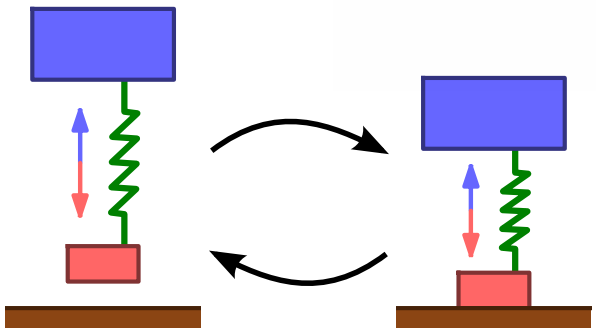


Discontinuities in vertical hopping

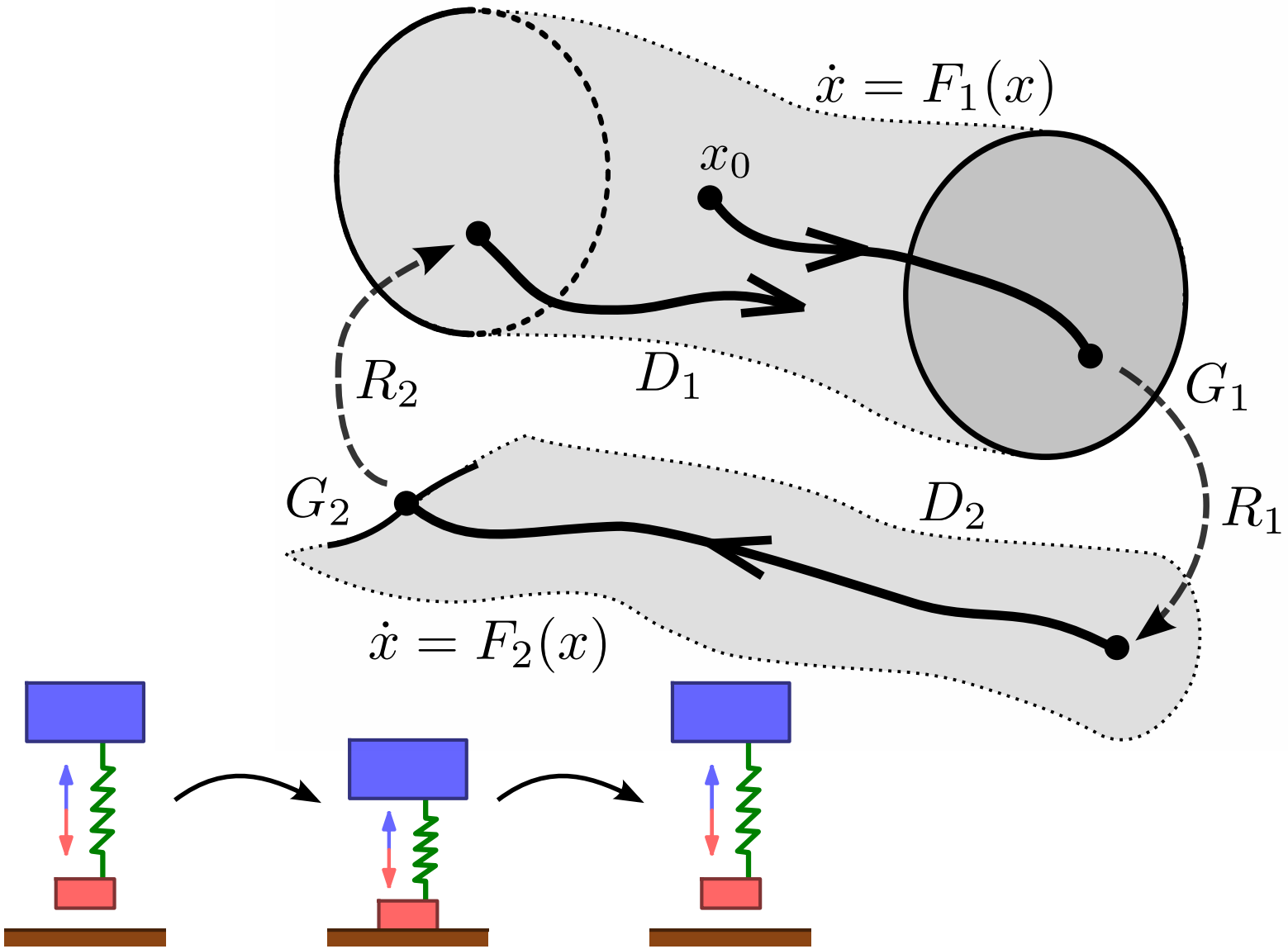


Parsimonious model for hopping is piecewise–defined



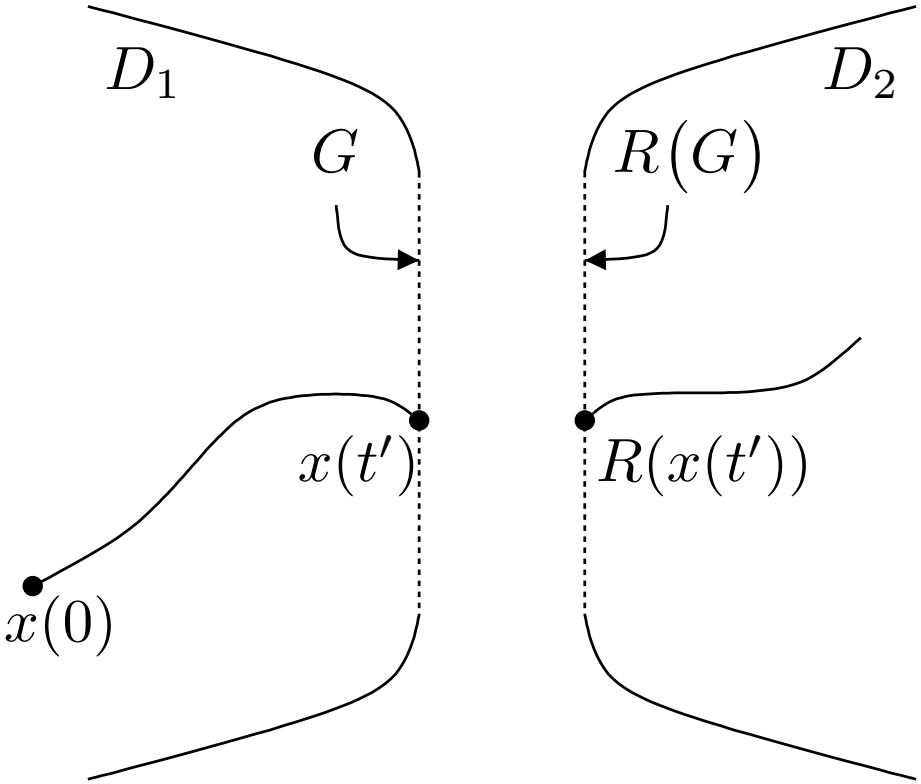


Trajectories are discontinuous at touchdown & liftoff



Mathematical "glue" removes discontinuities

disjoint state space D_1, D_2



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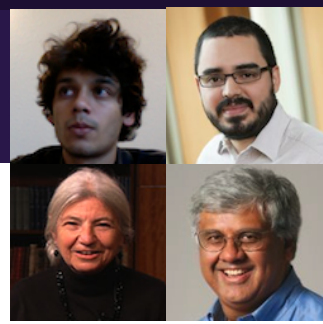
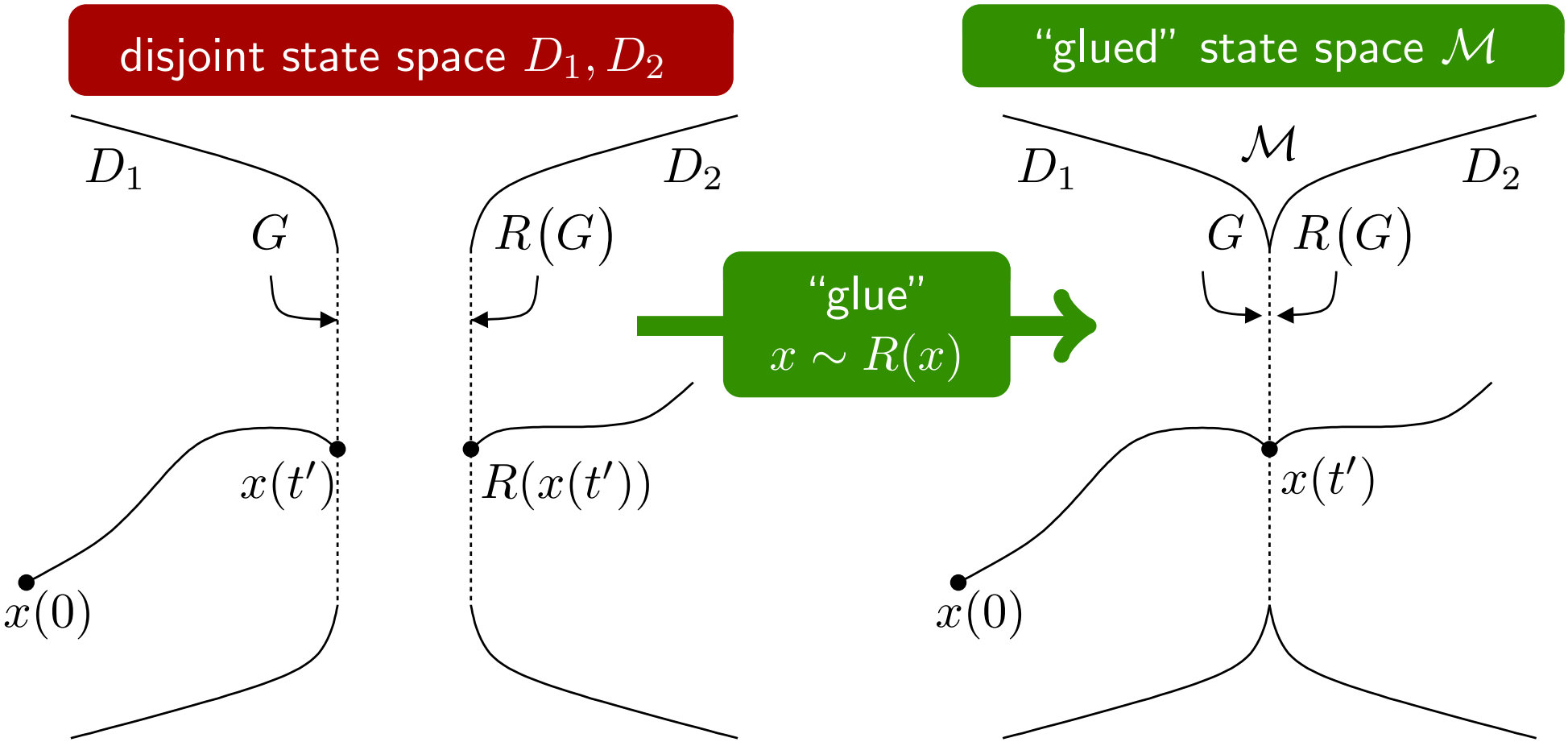
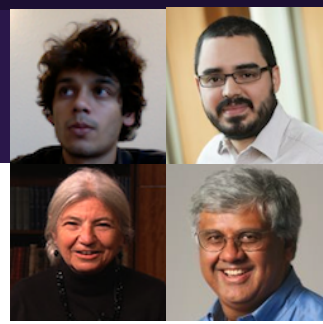




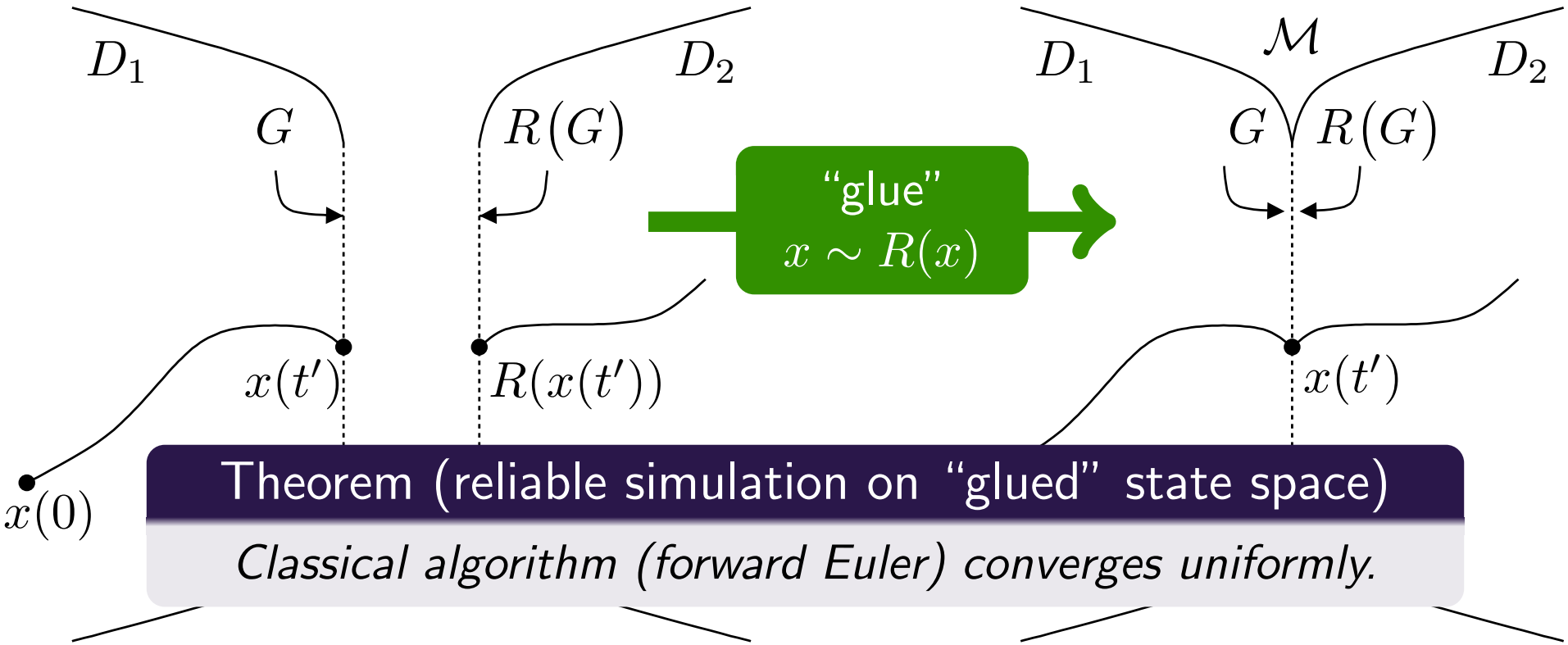
Burden, Gonzalez, Vasudevan, Bajcsy, Sastry (IEEE TAC 2015; arXiv:1302.4402)
Metrization and simulation for controlled hybrid systems

Mathematical "glue" removes discontinuities



disjoint state space D_1, D_2

"glued" state space $\mathcal{M}$

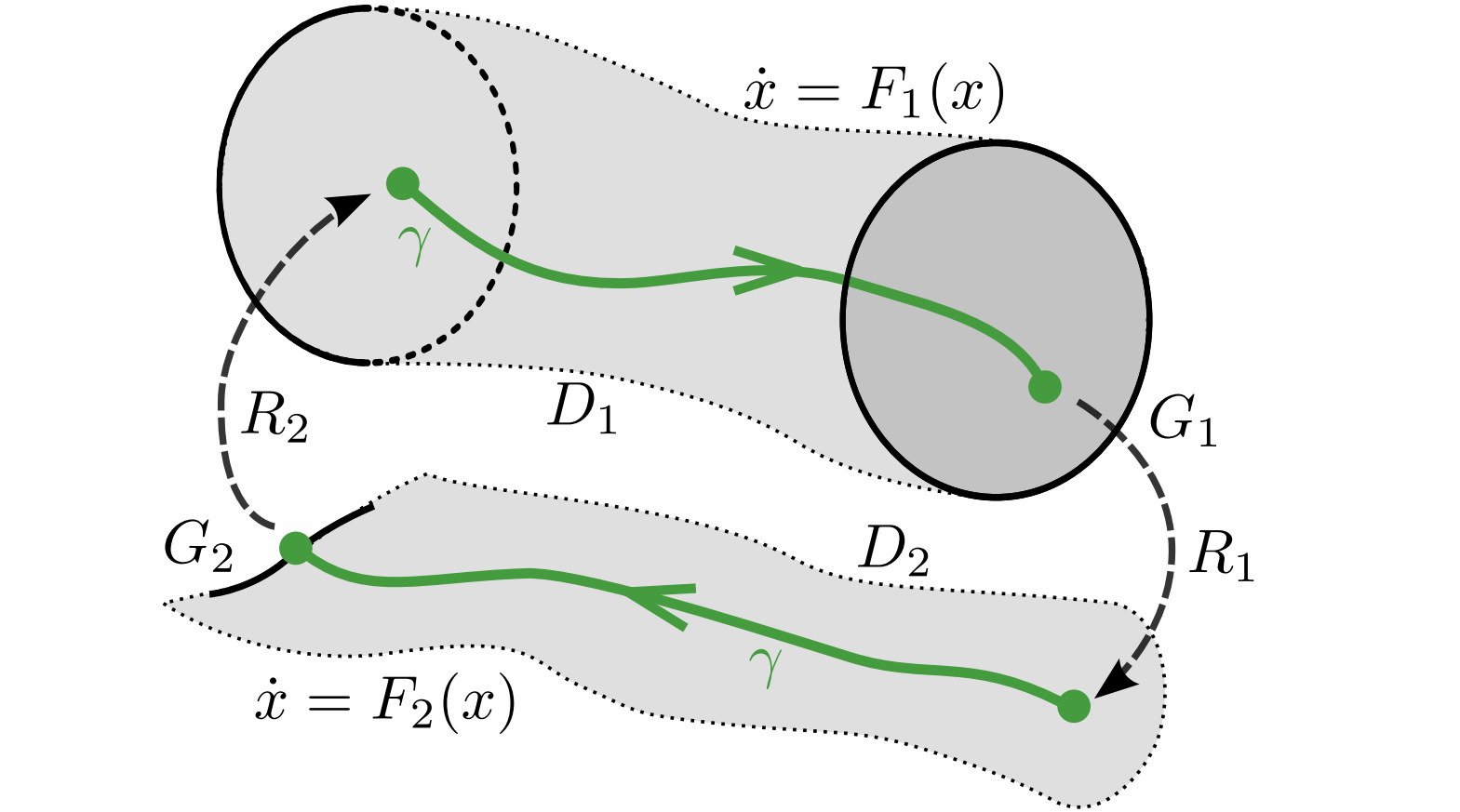


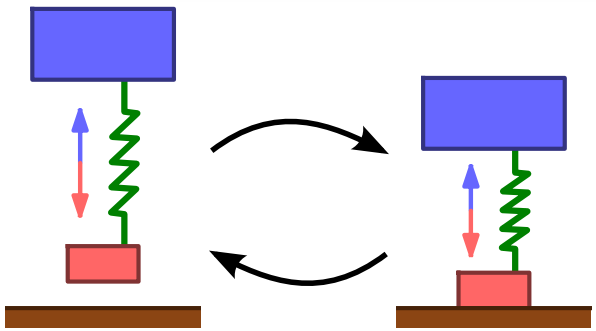
Theorem (reliable simulation on "glued" state space)

Classical algorithm (forward Euler) converges uniformly.

Burden, Gonzalez, Vasudevan, Bajcsy, Sastry (IEEE TAC 2015; arXiv:1302.4402)
Metrization and simulation for controlled hybrid systems

Novel reduction mechanism near periodic orbit γ





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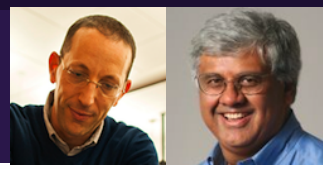
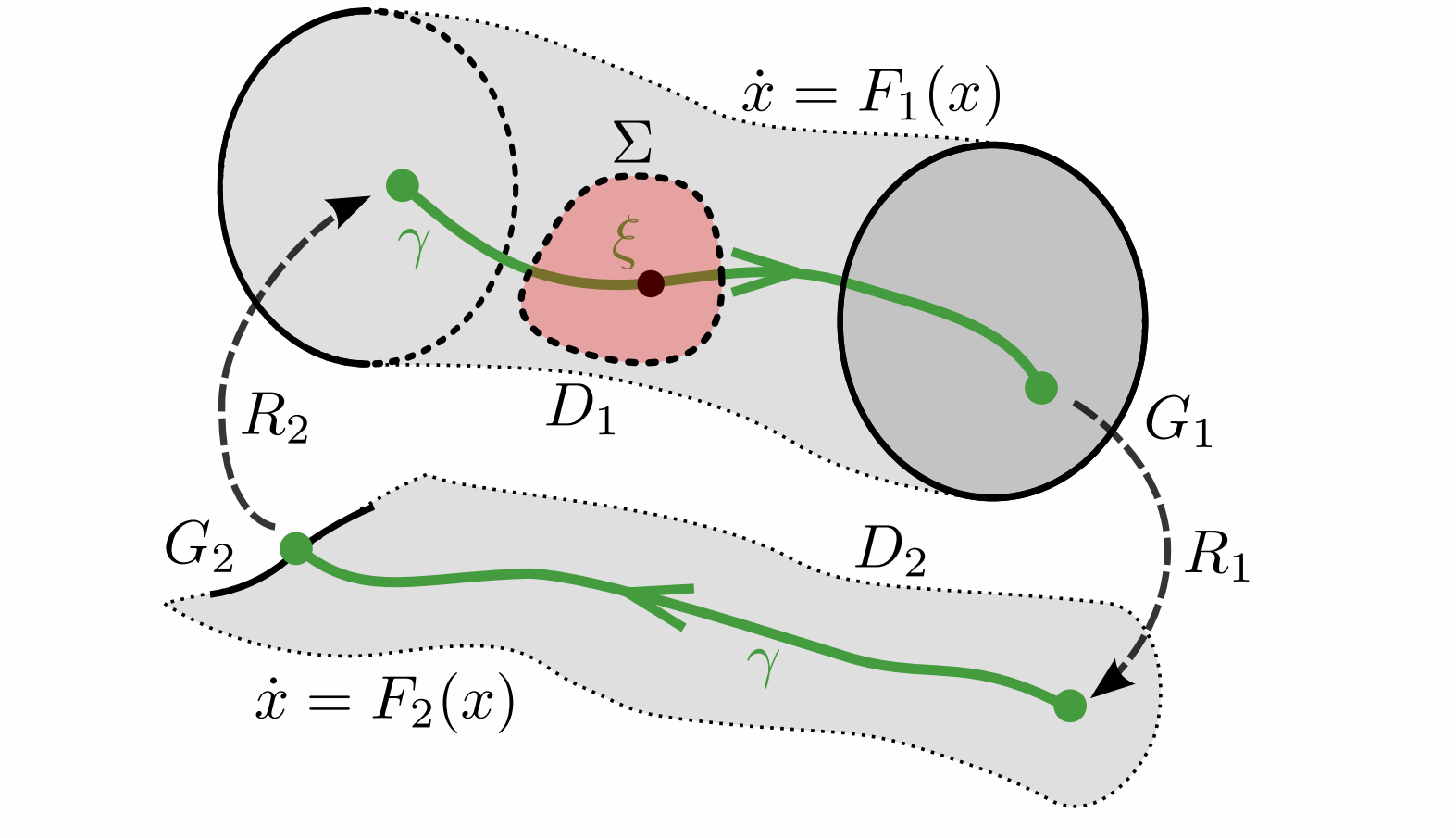
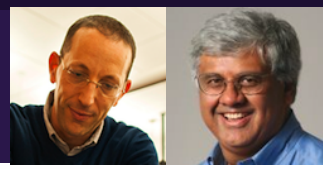
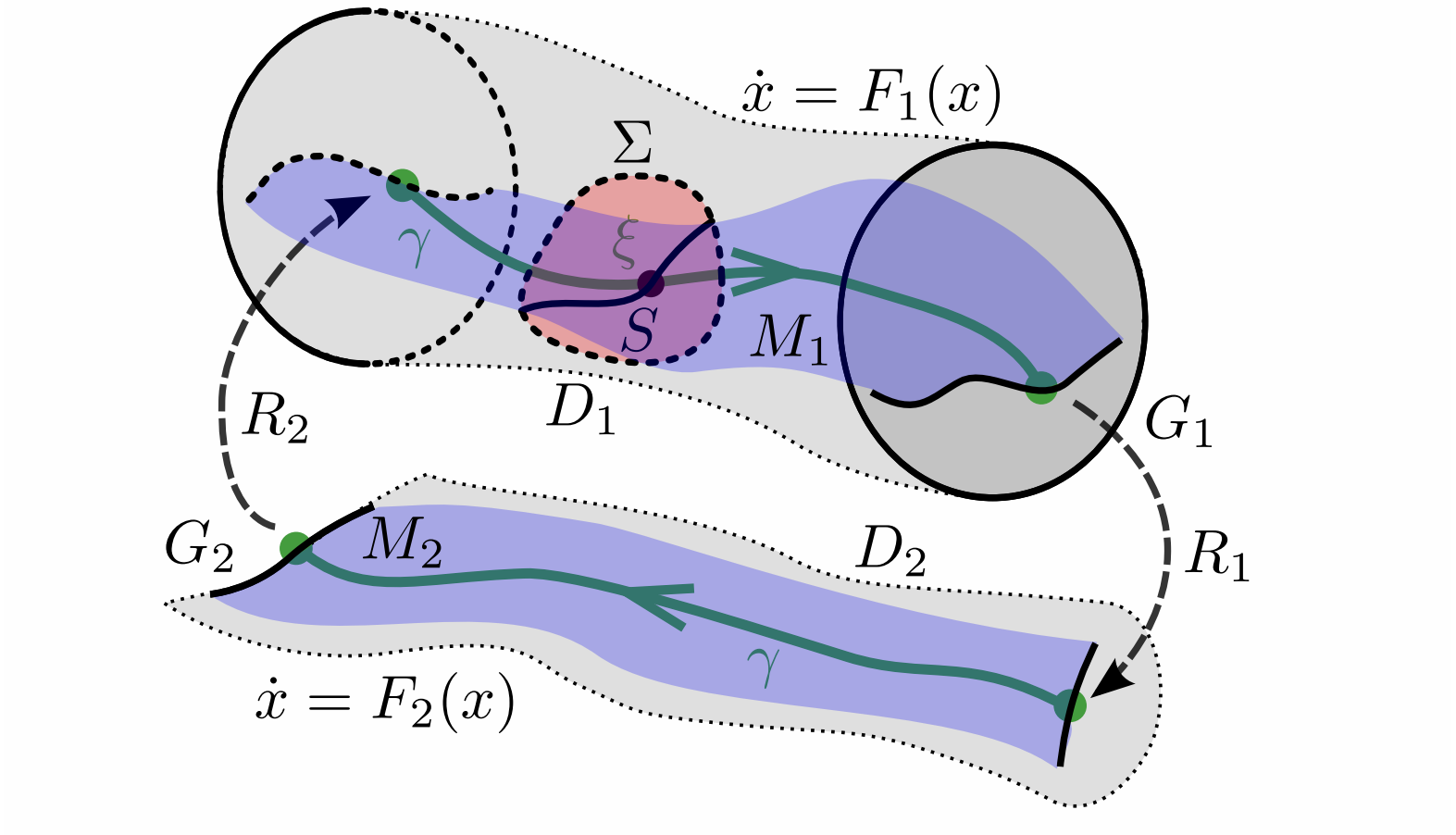




Burden, Revzen, Sastry (IEEE TAC 2015; arXiv:1308.4158)
Model reduction near periodic orbits of hybrid dynamical systems

Novel reduction mechanism near periodic orbit γ





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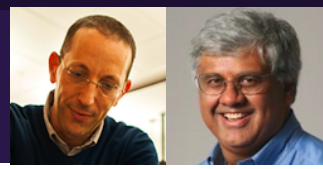

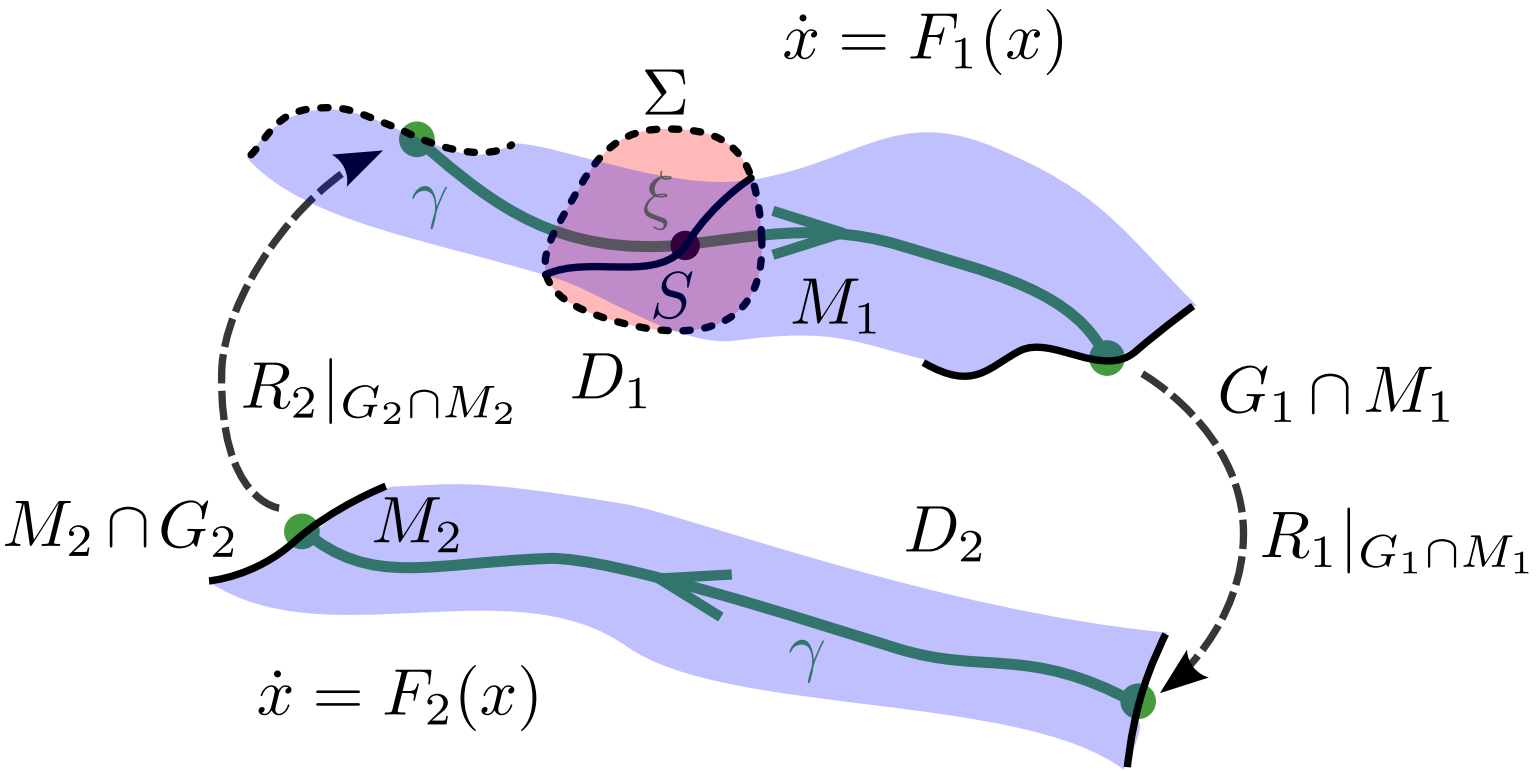

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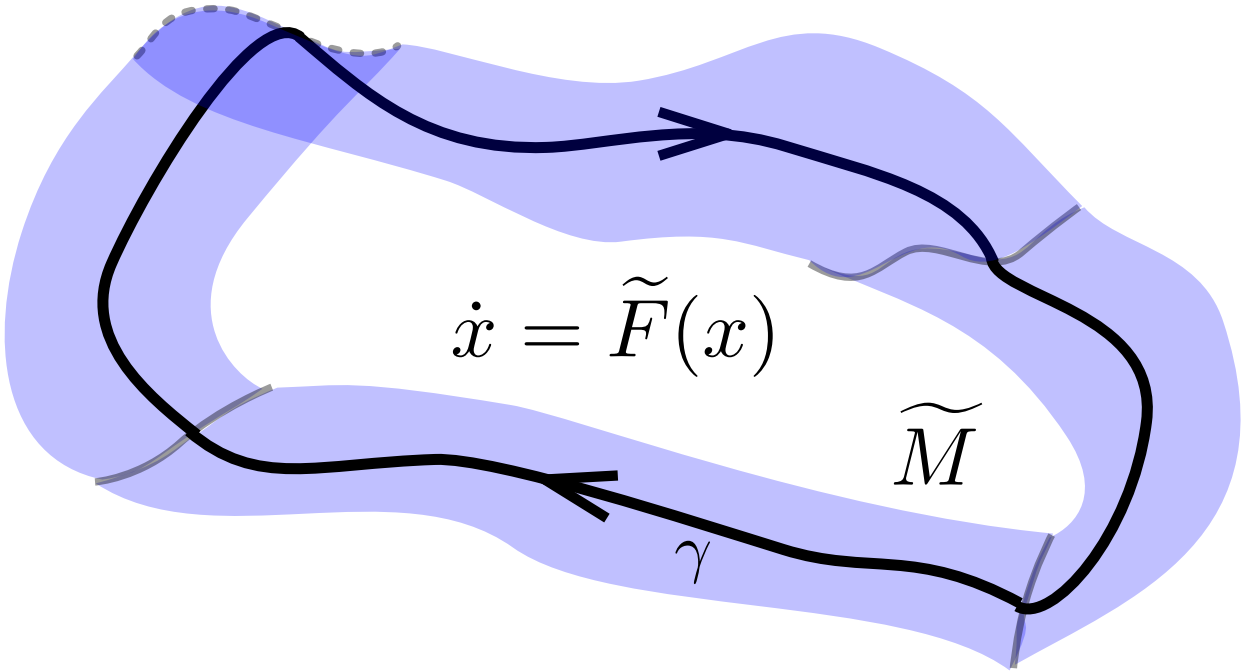

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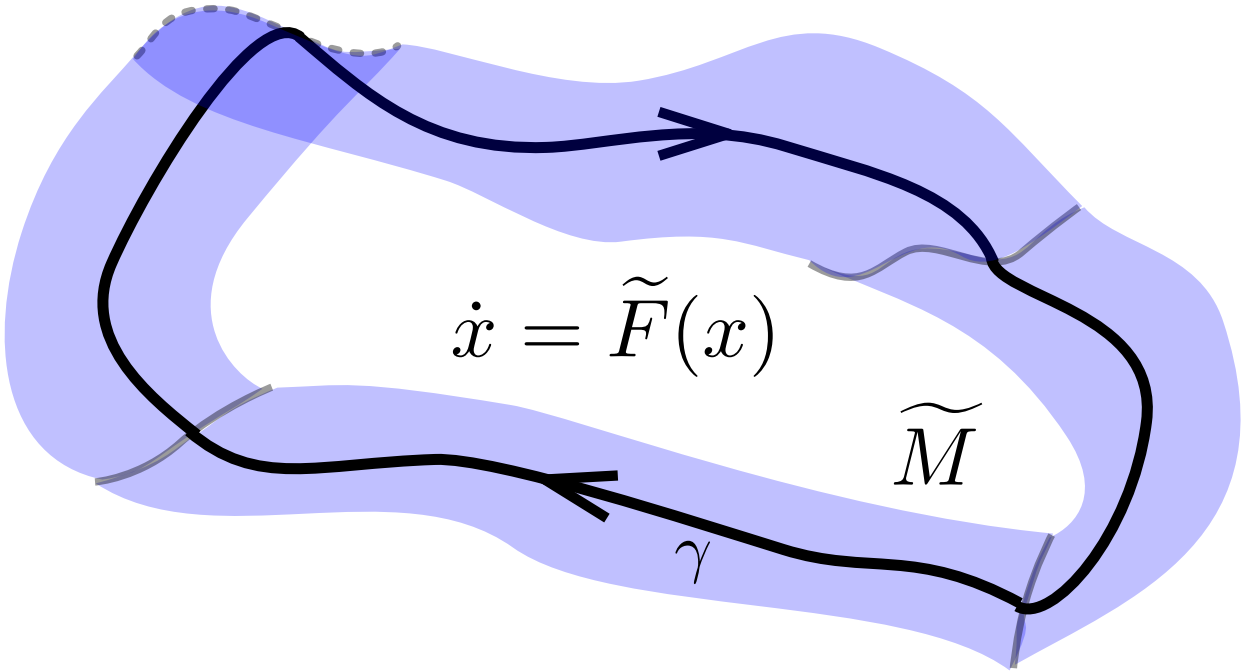

Theorem (model reduction near periodic orbit)

Piecewise–defined model reduces to smooth differential equation on "glued" space.

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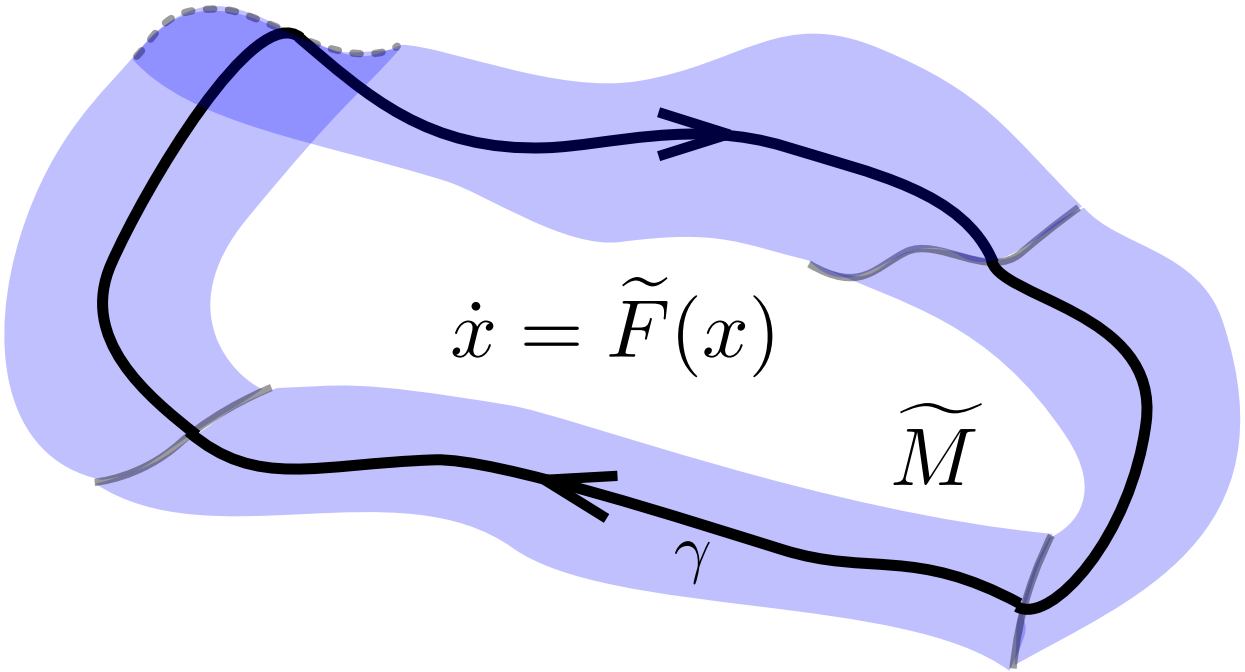
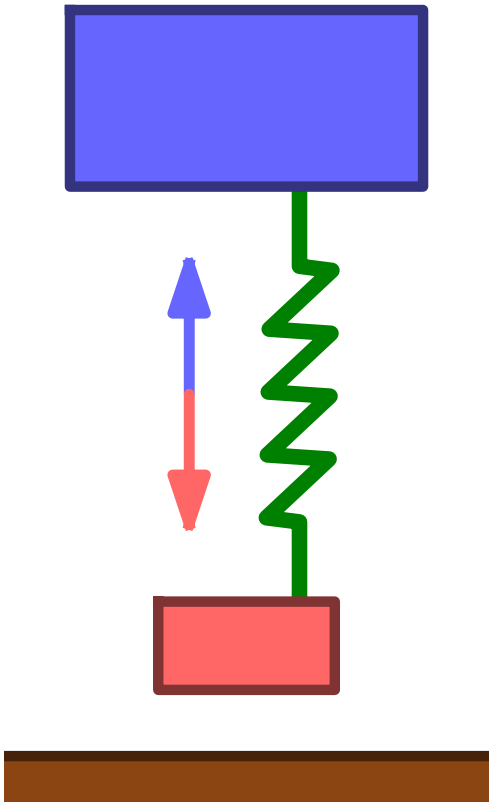




Theorem (model reduction near periodic orbit)

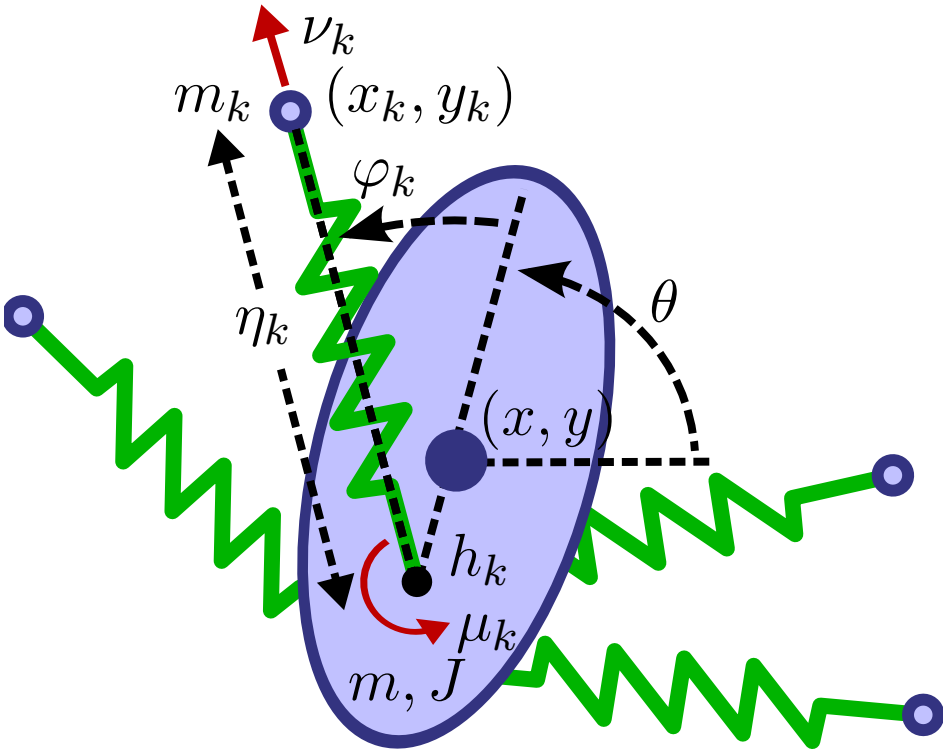
Piecewise–defined model reduces to smooth differential equation on "glued" space.

Example (hopper)

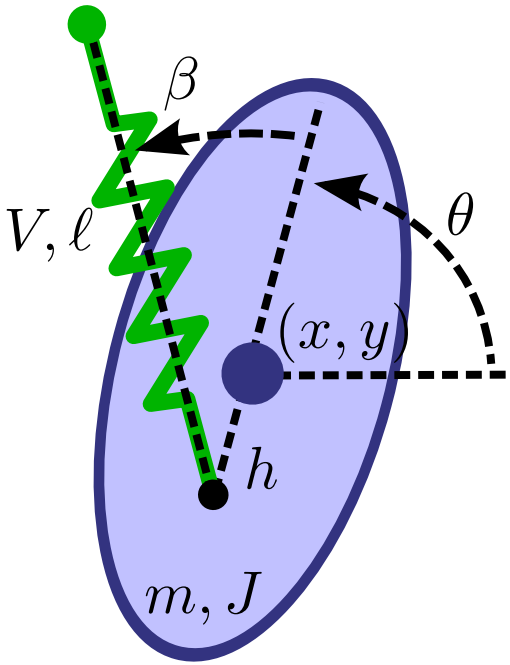
Reduces to smooth 2D subsystem.

Burden, Revzen, Sastry (IEEE TAC 2015; arXiv:1308.4158)
Model reduction near periodic orbits of hybrid dynamical systems

Model with n legs reduces to Lateral Leg–Spring



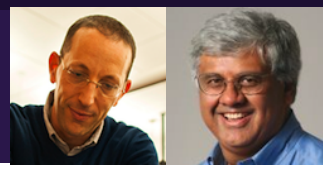
$(3 + 2n)$ degrees–of–freedom

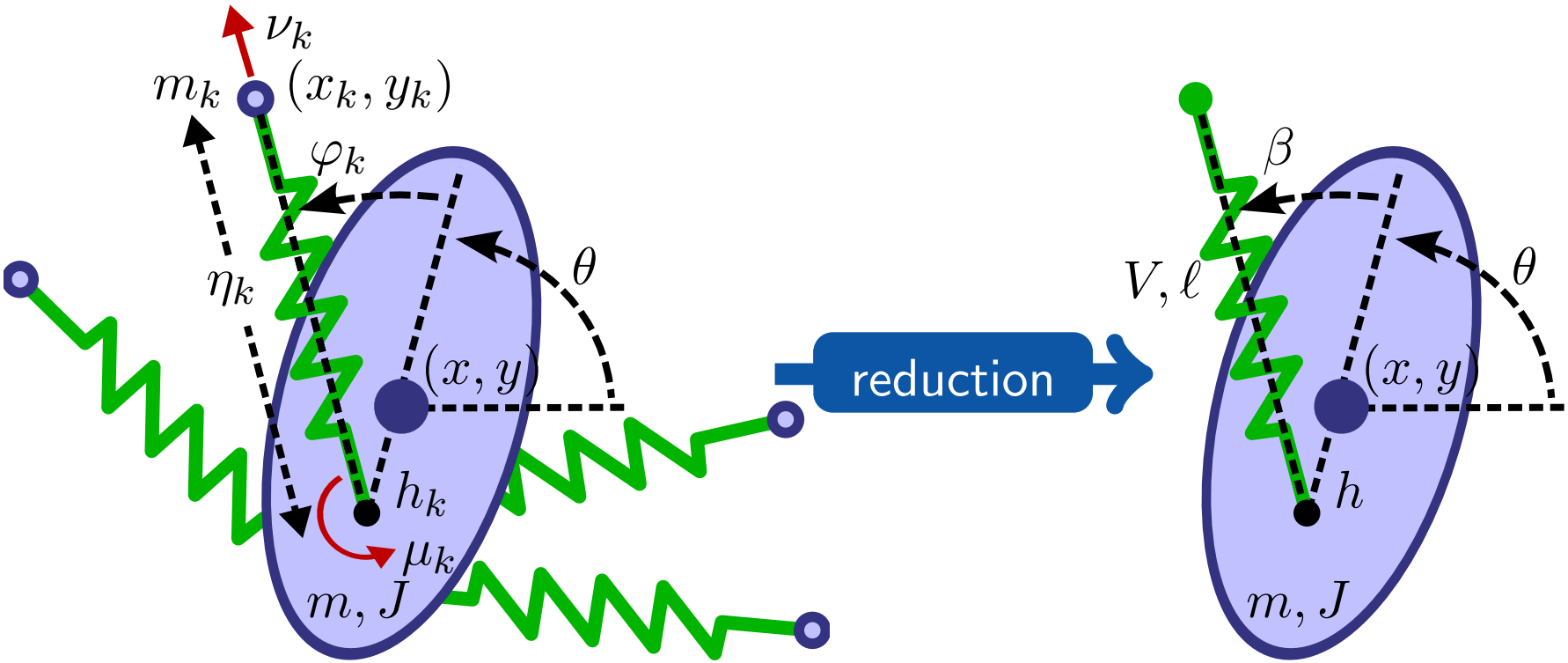


3 degrees–of–freedom

Schmit & Holmes 2001

Model with n legs reduces to Lateral Leg–Spring





$(3 + 2n)$ degrees–of–freedom

3 degrees–of–freedom

Schmit & Holmes 2001

Controller (Burden, Revzen, Sastry IEEE TAC 2015; arXiv:1308.4158)

Smooth feedback law reduces $2n$ degrees–of–freedom after one stride.

Contribution from removal of discontinuities

Motivation: legged locomotion involves *intermittent* interaction
Predictions limited by pathologies in parsimonious models.

1. Mathematical "glue" removes discontinuities
   Yields reliable simulation algorithm and novel route to model reduction.

2. Restricting impact restitution resolves inconsistencies
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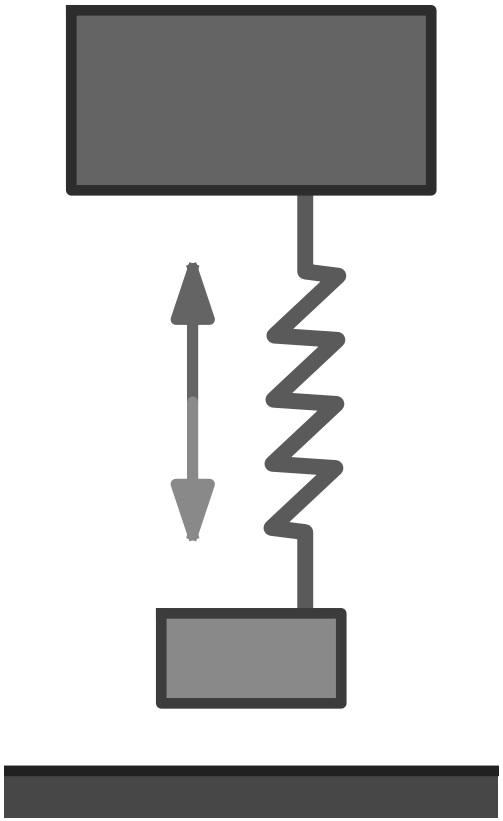
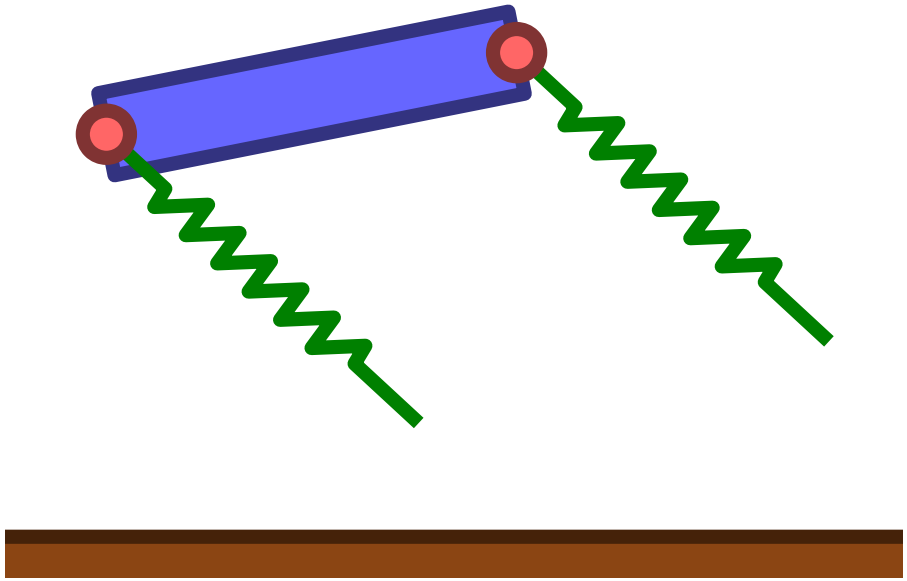
Future directions: predictions for robotics & biology
Effect of parameters and perturbations on gaits and maneuvers.

Parsimonious predictive models for legged locomotion



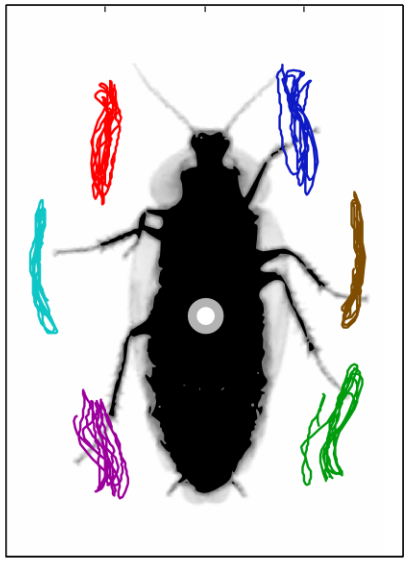


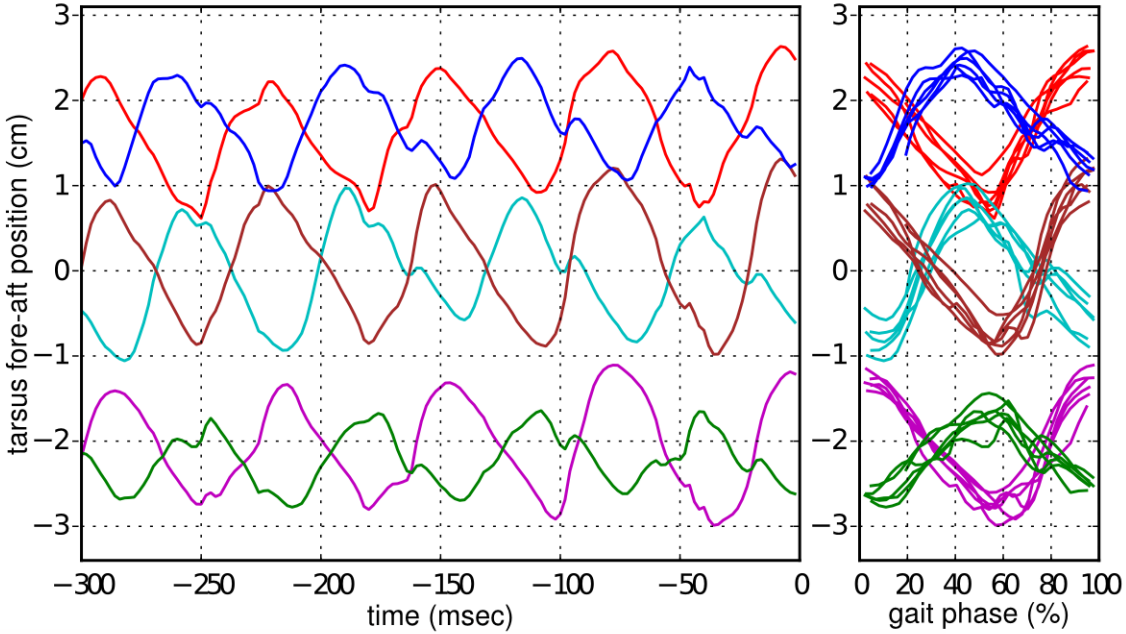
1. Remove discontinuities

2. Resolve inconsistencies

Near–simultaneous limb touchdown in animal gaits

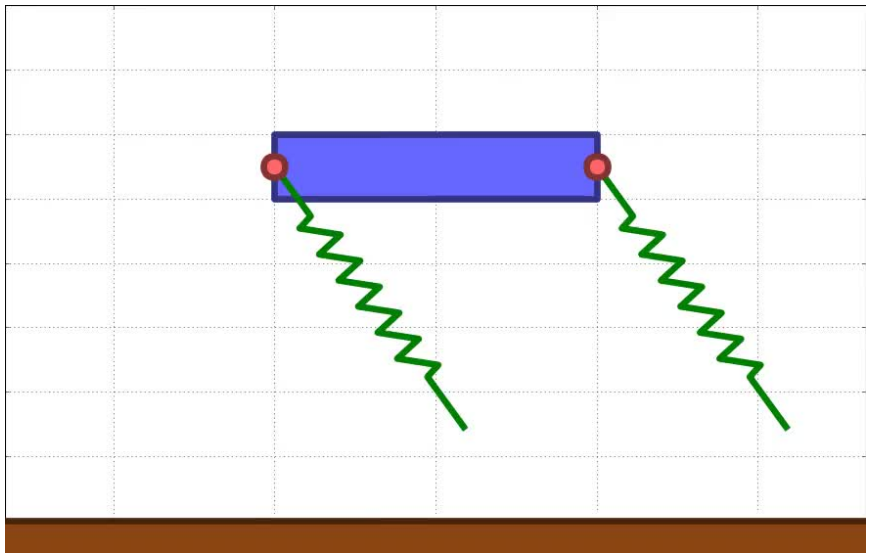
alternating tripod





MeMyHorseAndI.com

trot

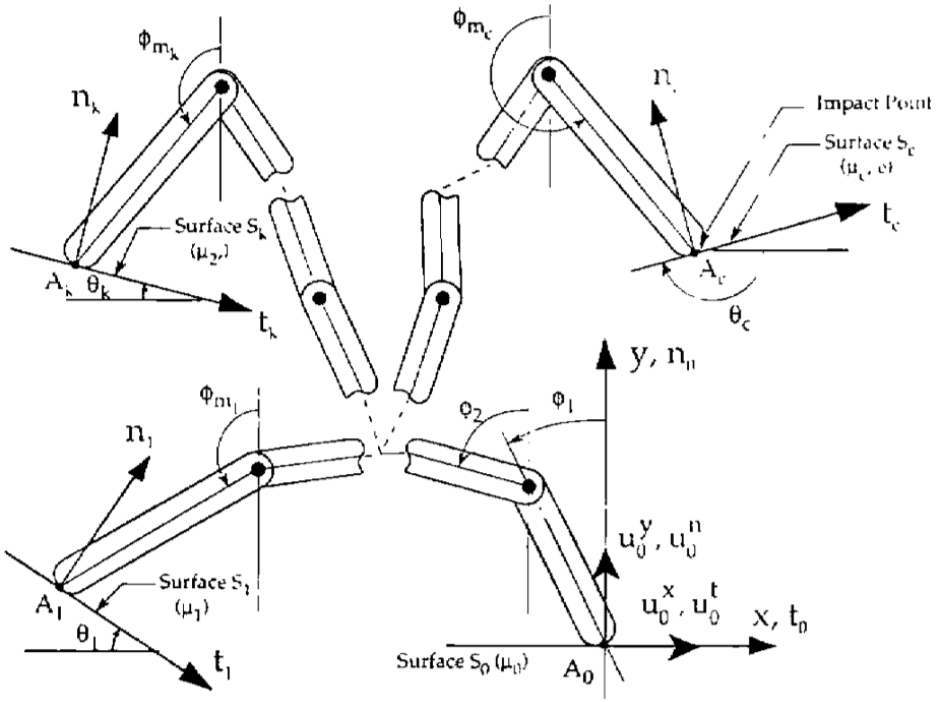


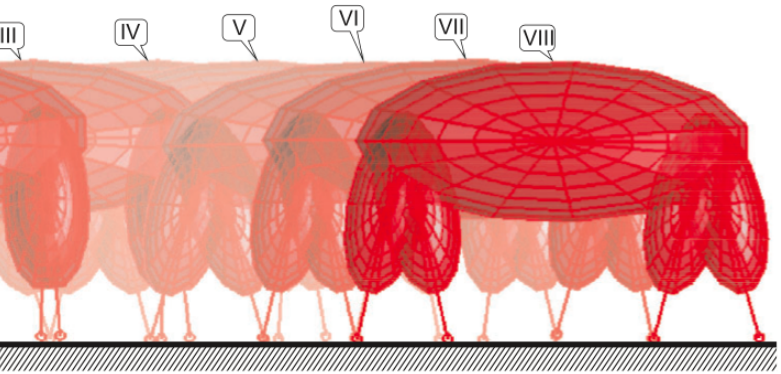
Near–simultaneous limb touchdown in robot gaits

Galloway, Haynes, Ilhan, Johnson, Knopf, Lynch, Plotnick, White, Koditschek UPenn 2010

Hyun, Seok, Lee, Kim IJRR 2014

Rigidity leads to inconsistencies near simultaneous impact





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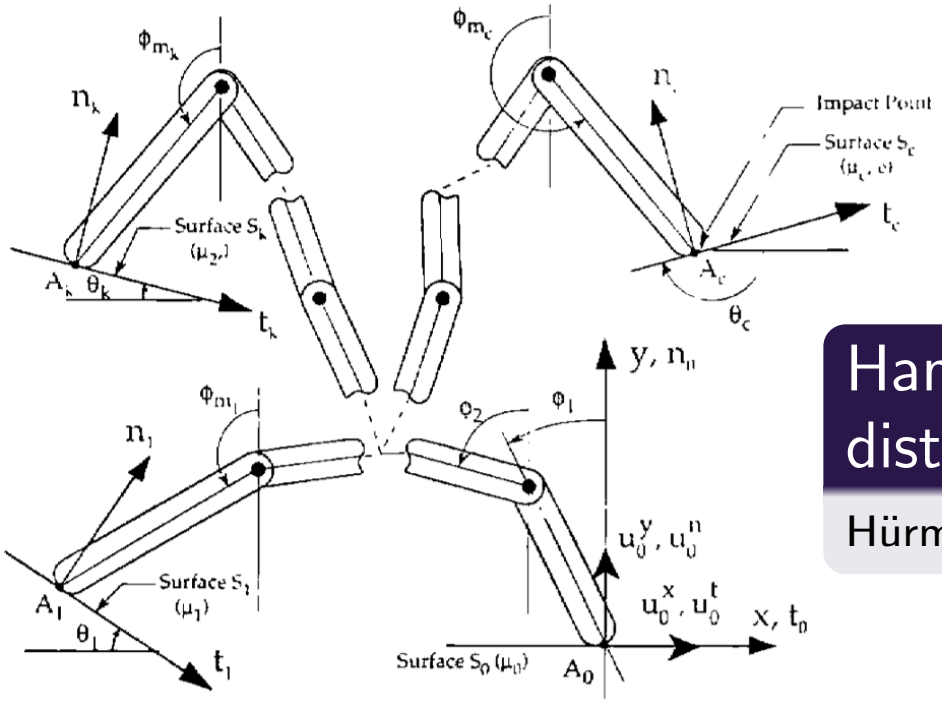

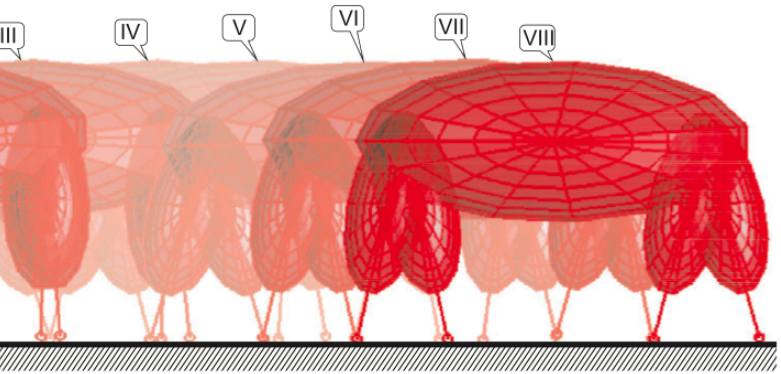

Hands with rigid fingers admit 5 (!) distinct outcomes after grasp

Hürmüzlü and Marghitu IJRR 1994, JAM 1995

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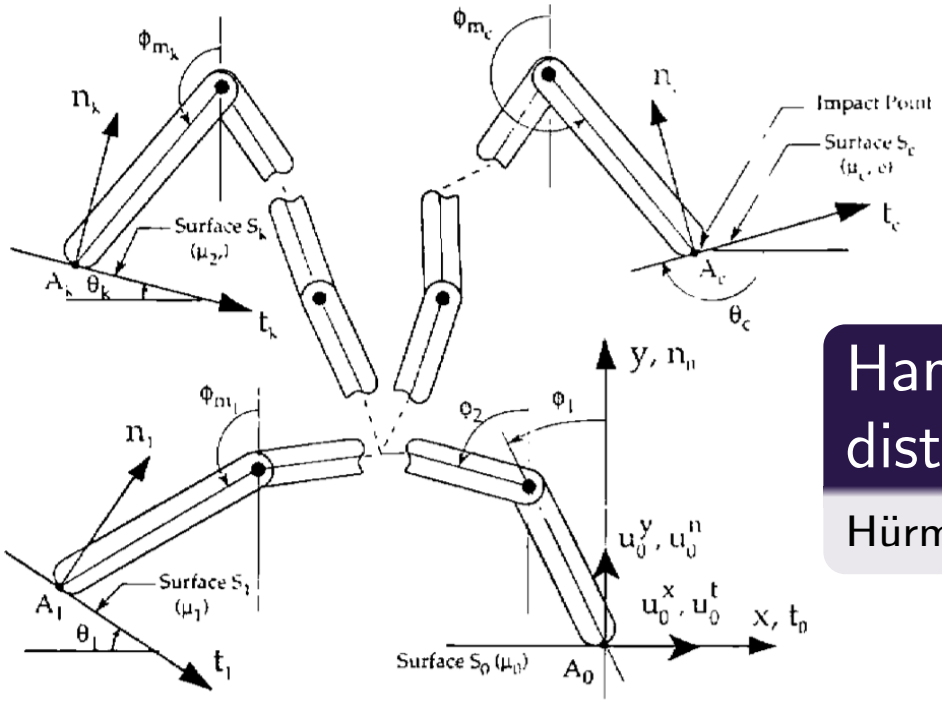

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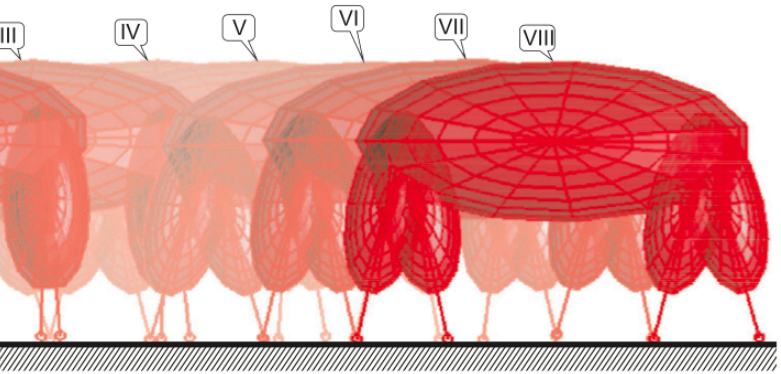
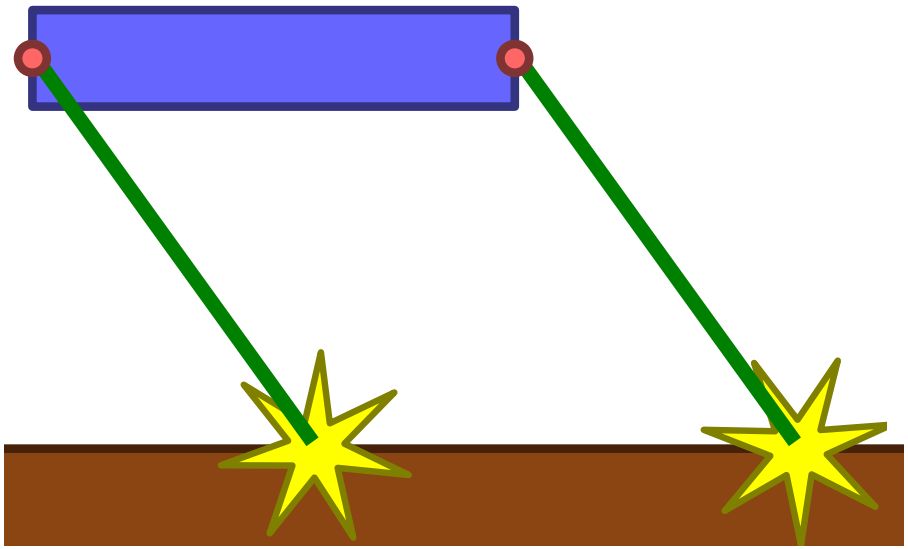


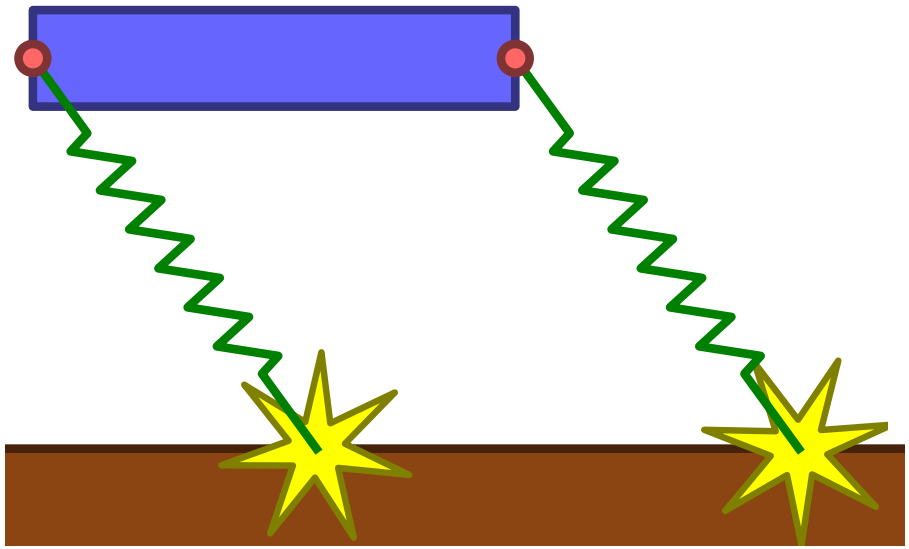
Quadruped with rigid legs possesses three distinct trot gaits

Remy, Buffington, Siegwart IJRR 2010

Restricting impact restitution resolves inconsistencies



Rigid limbs



Viscoelastic limbs

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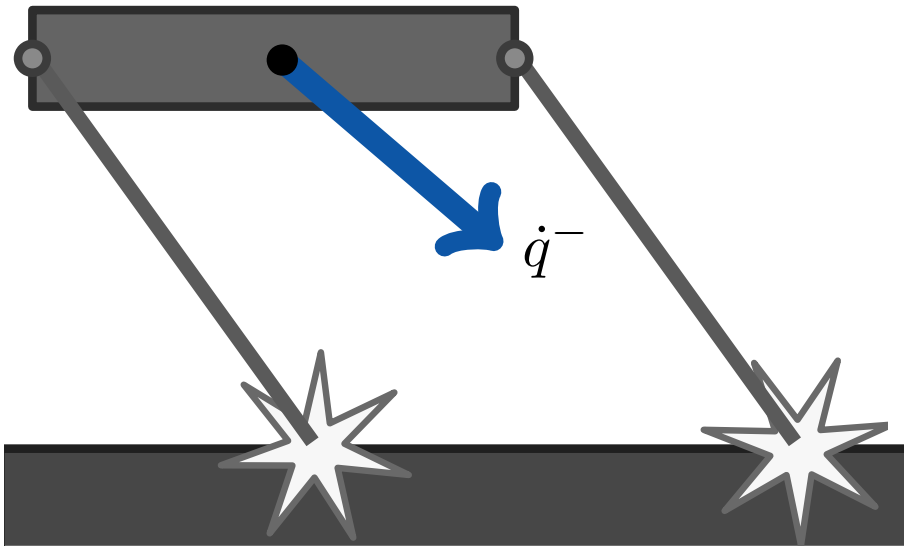
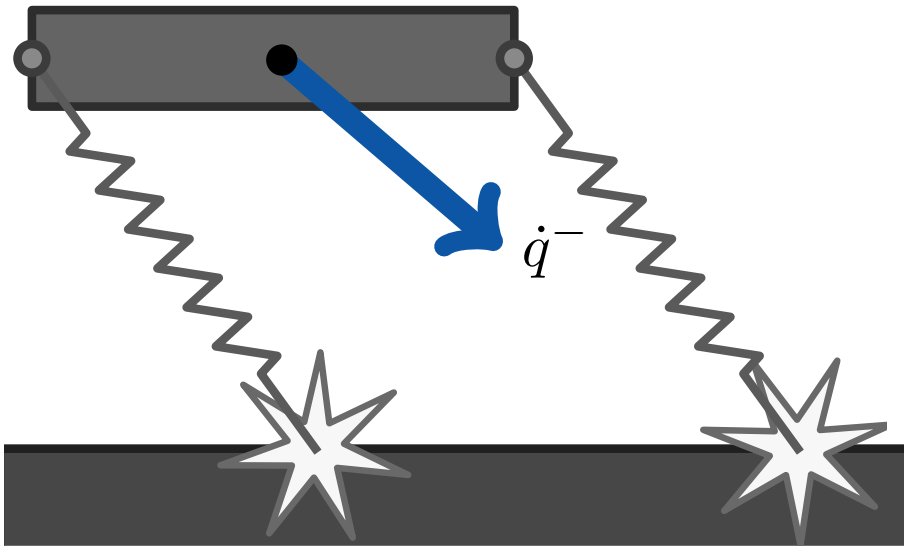


Rigid limbs



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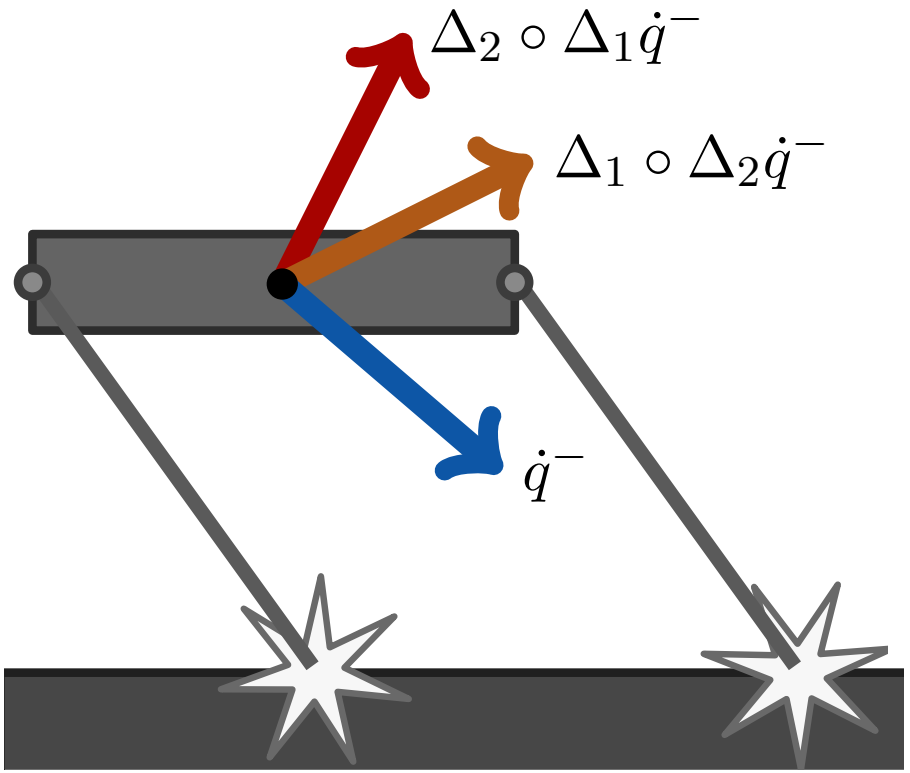

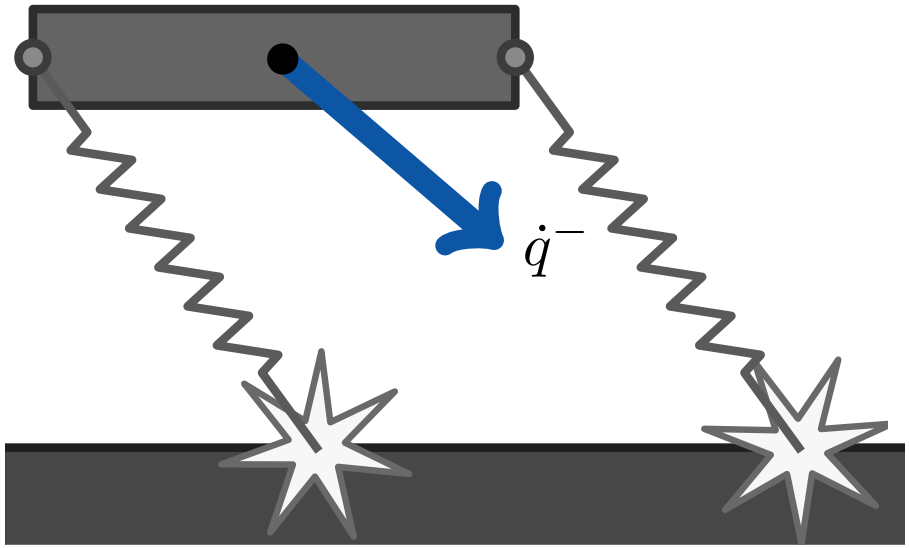

Rigid limbs

$$\Delta_1 \circ \Delta_2 \dot{q}^- \neq \Delta_2 \circ \Delta_1 \dot{q}^-$$

Viscoelastic limbs

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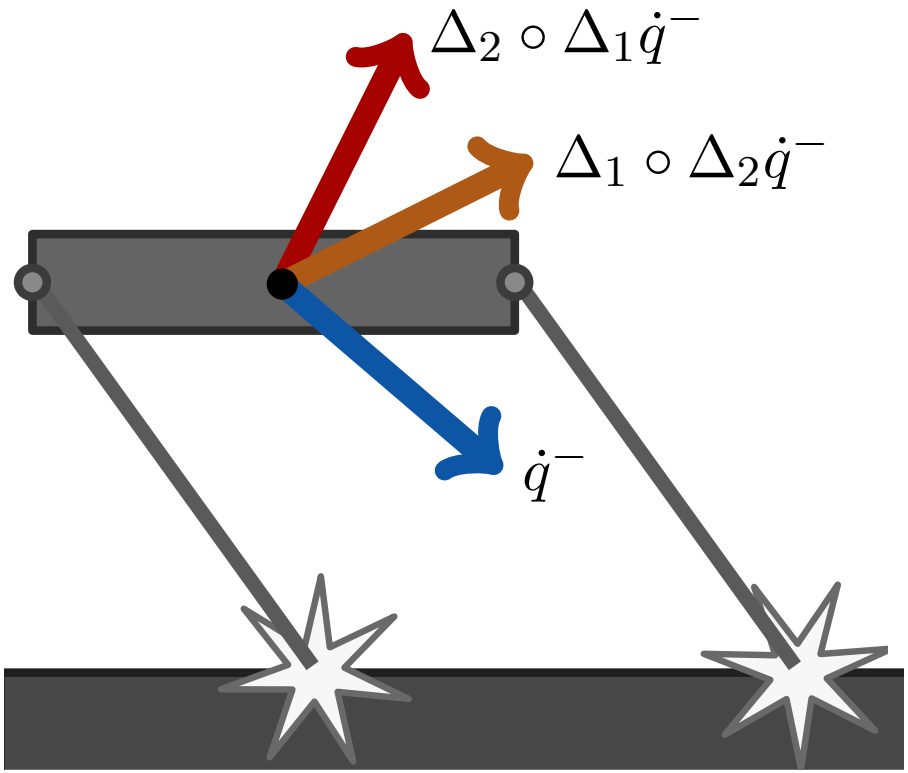

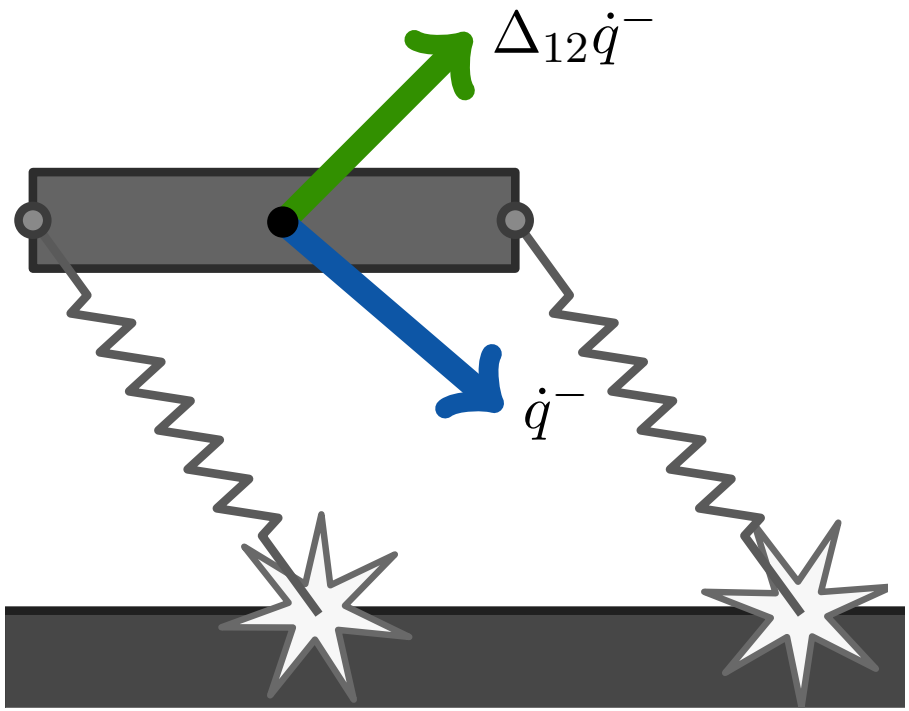

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Viscoelastic limbs

$$\Delta_1 \circ \Delta_2 \dot{q}^- = \Delta_2 \circ \Delta_1 \dot{q}^- =: \Delta_{12} \dot{q}^-$$

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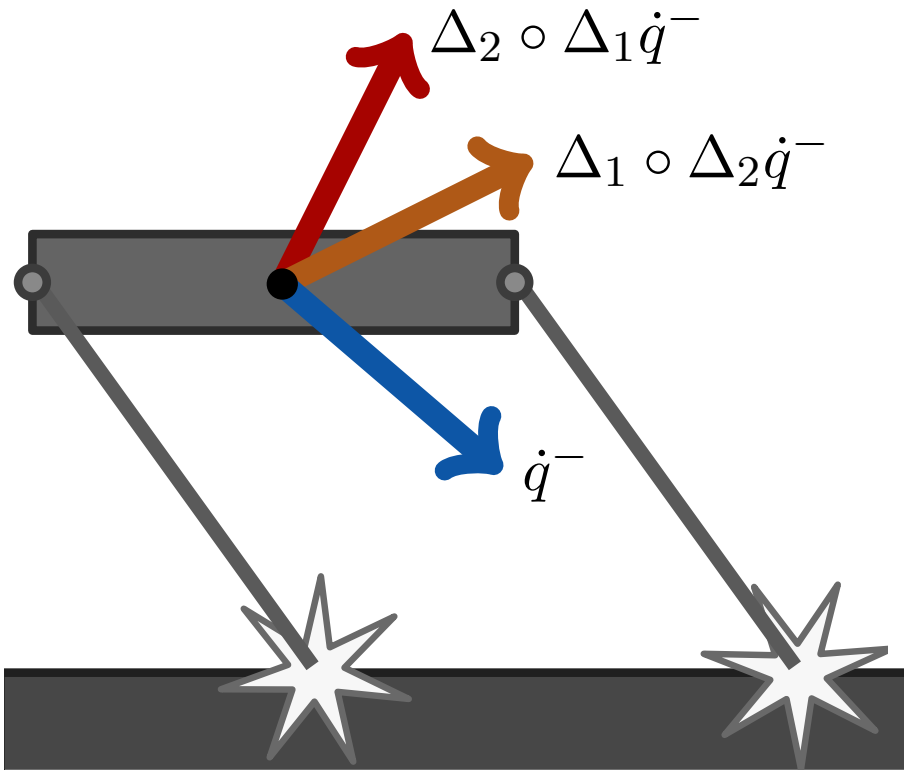

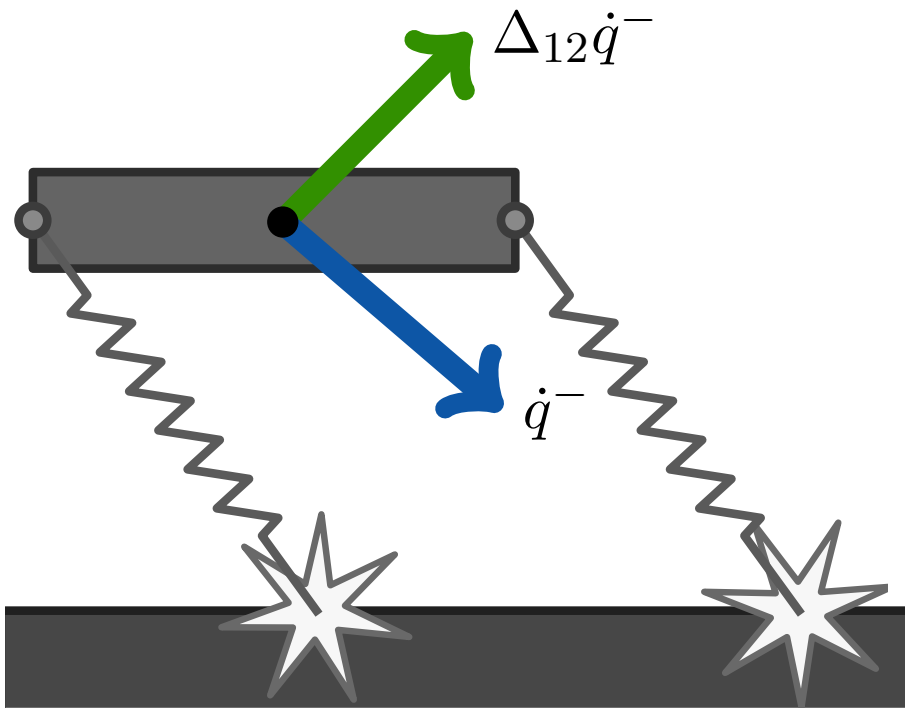

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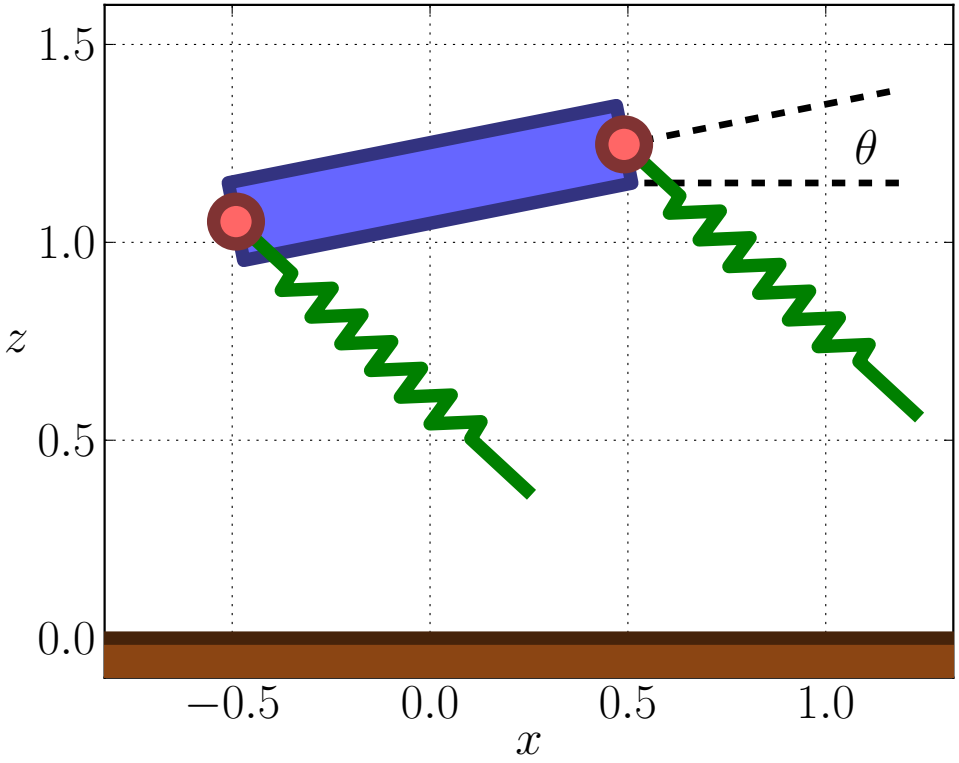
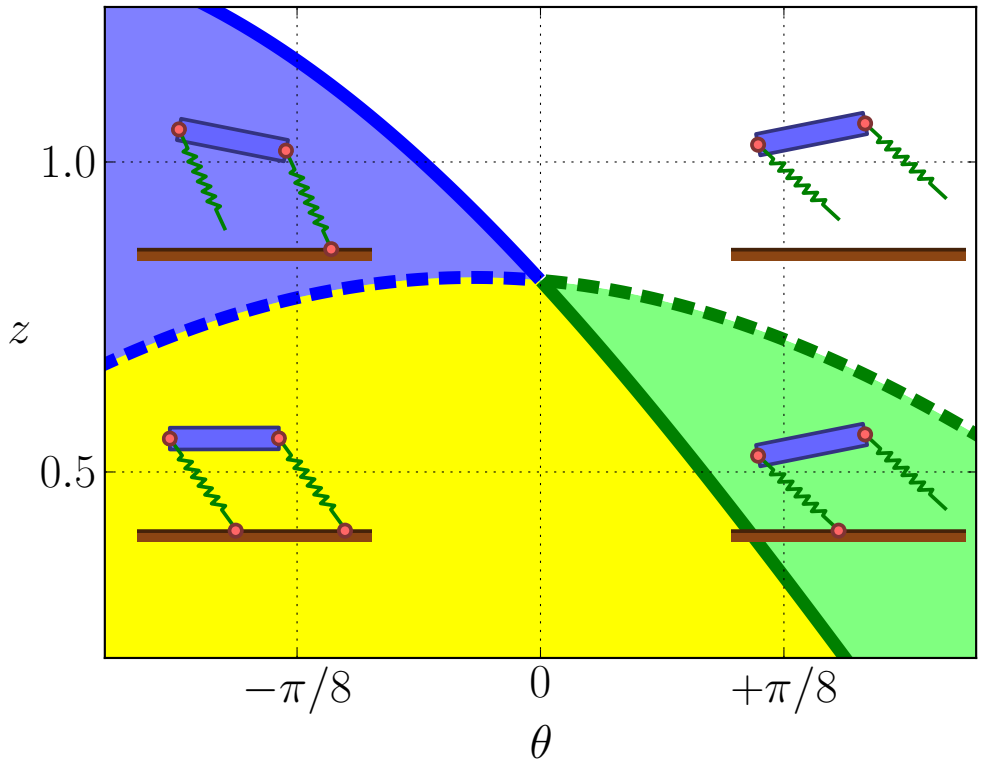
Viscoelastic limbs lead to consistent post–impact velocity.

Novel mechanism for rotation stabilization





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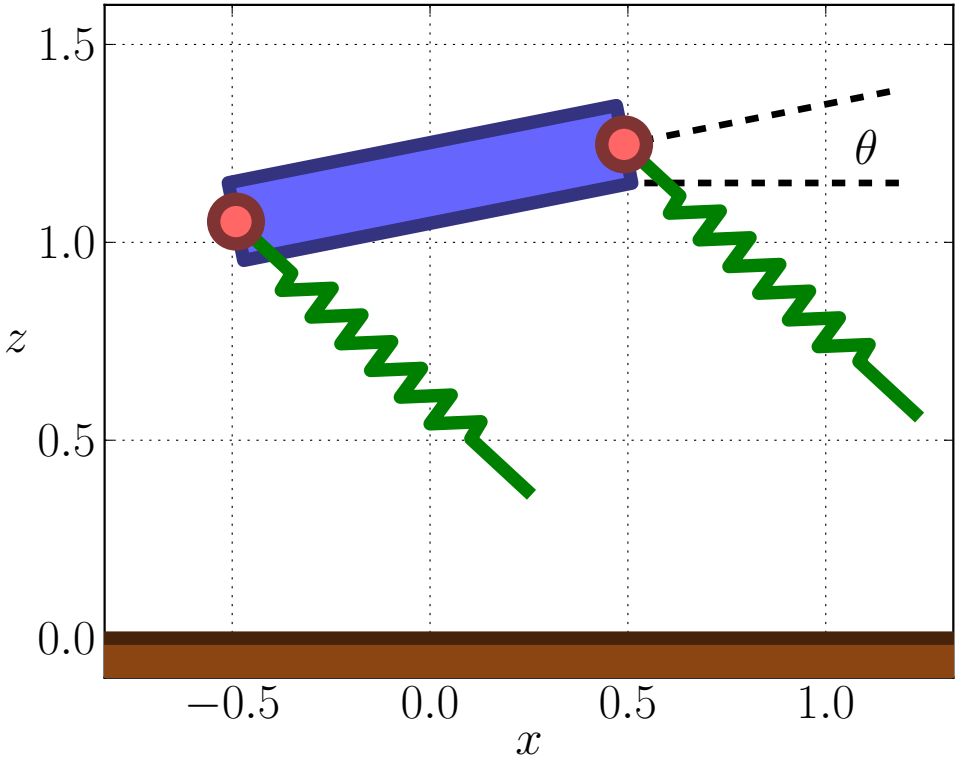

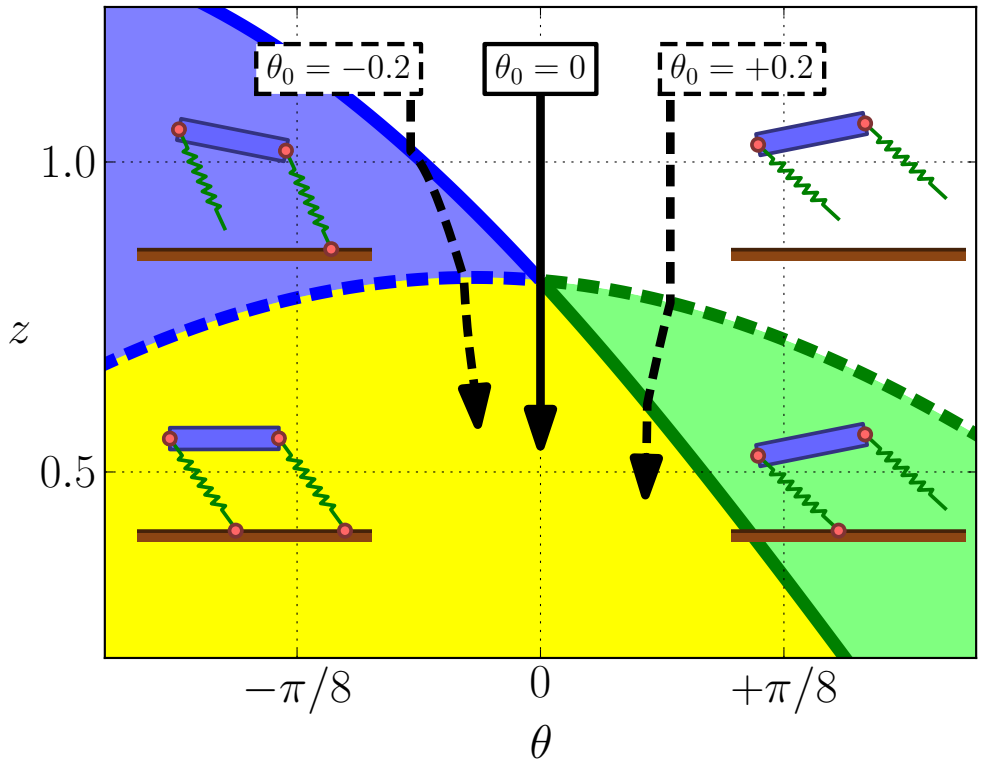

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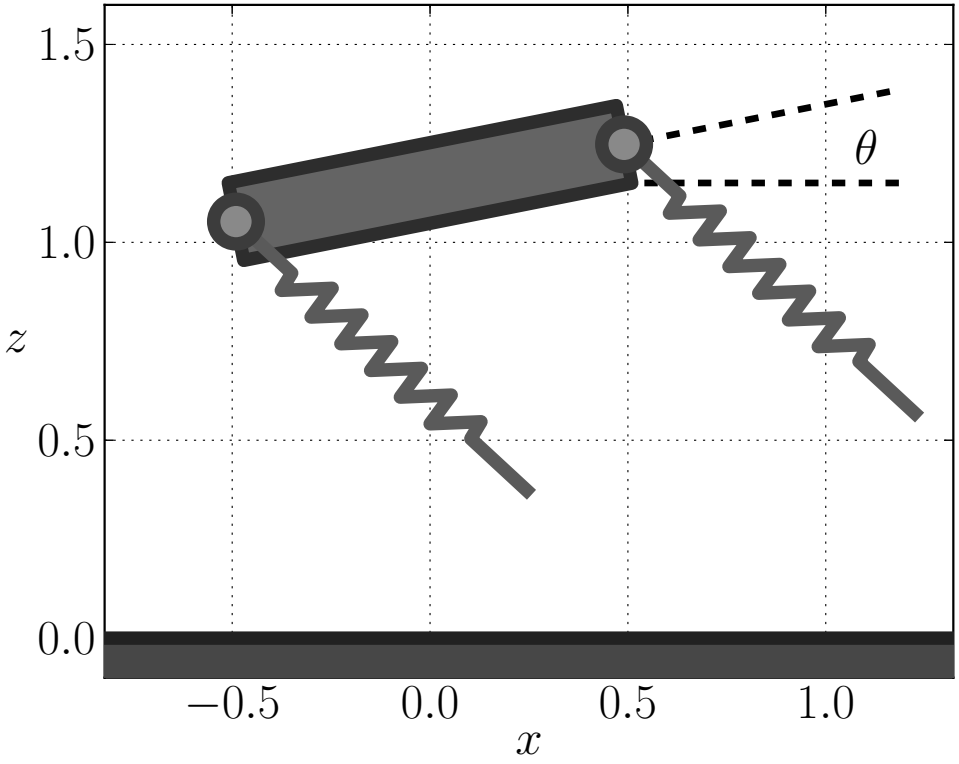

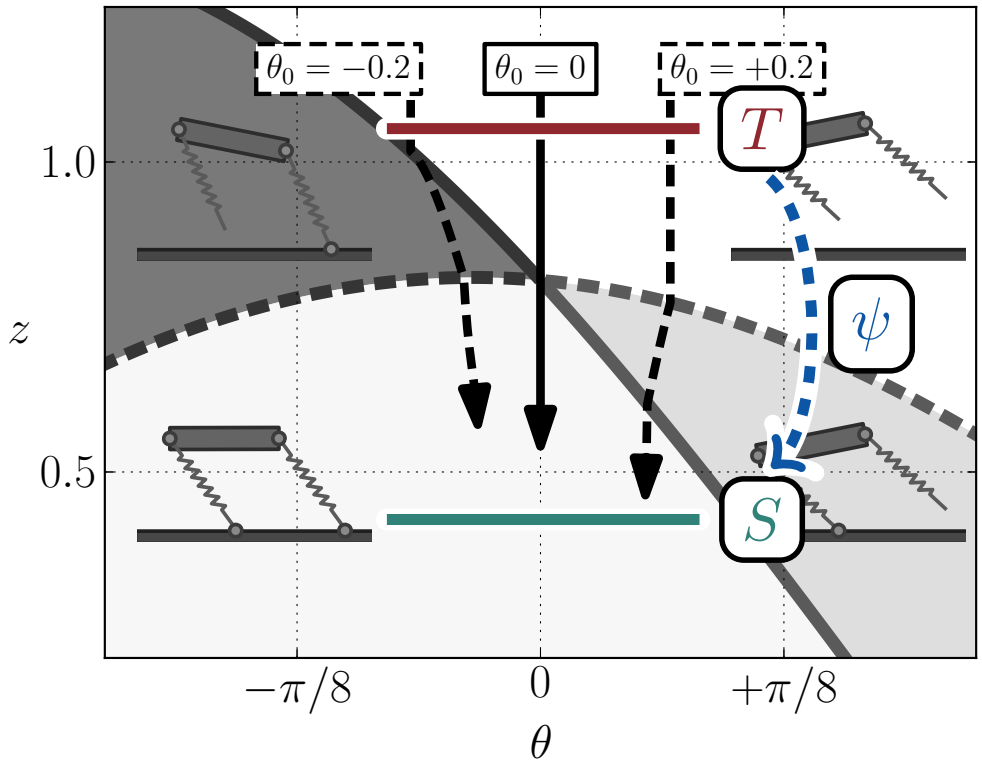

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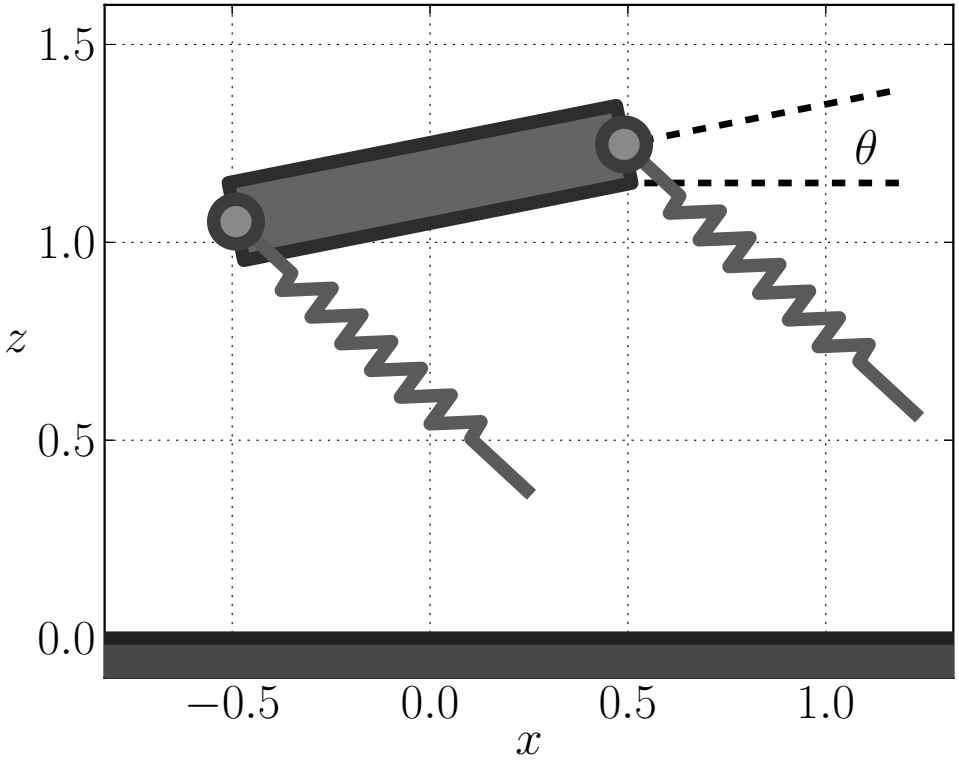
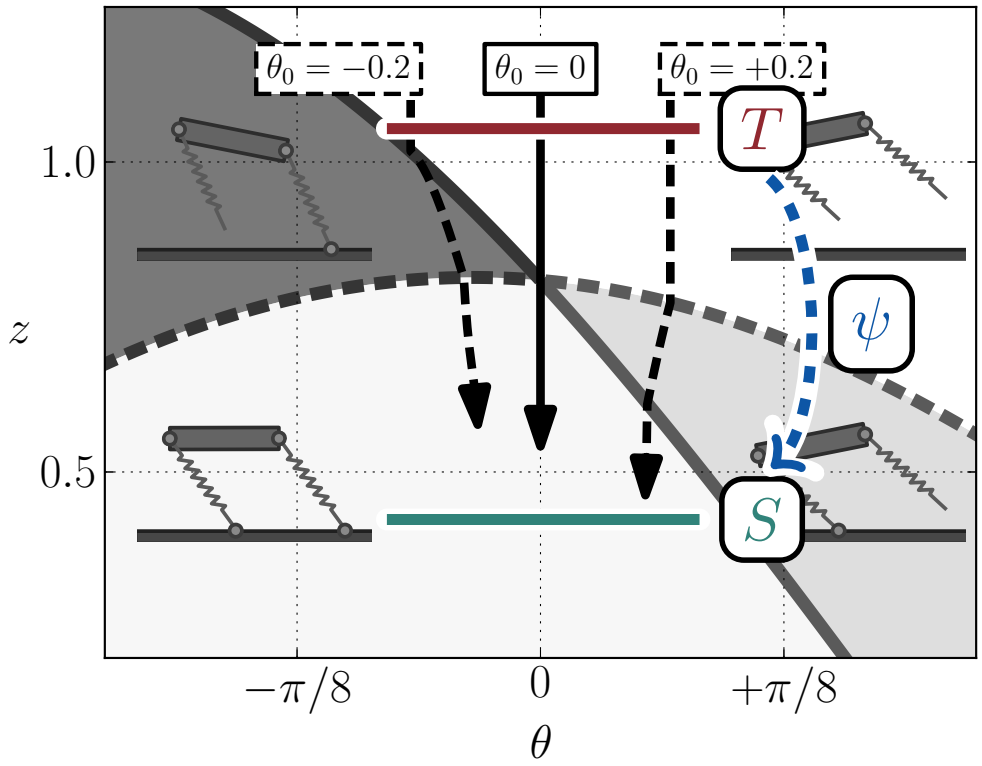
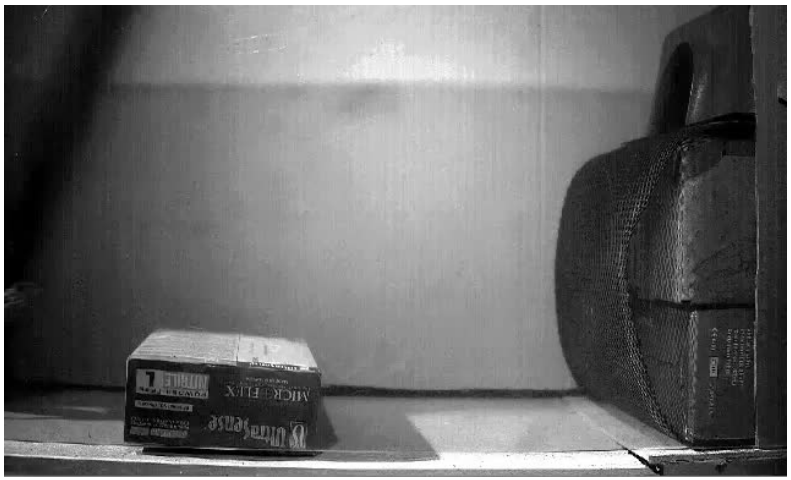




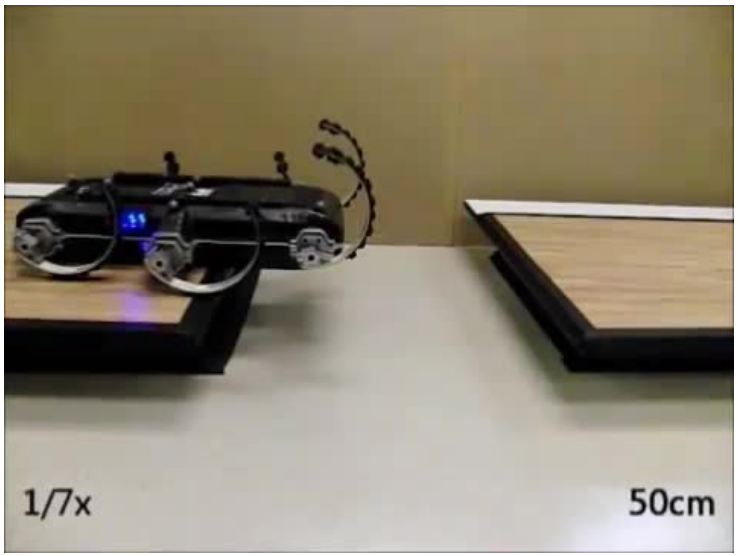
Viscoelastic limbs imply $\psi : T \to S$ contracts pitch

Near–simultaneous limb touchdown lends stability.

Resolving inconsistencies enables scalable optimization



Libby, Moore, Chang–Siu, Li, Cohen, Jusufi, Full Nature 2012



Johnson & Koditschek ICRA 2013

Synthesis of gaits, maneuvers

$$\min_{\tau} \quad J(q, \dot{q})$$

$$\text{s.t.} \quad \ddot{q} = f(q, \dot{q}) + \lambda(q, \dot{q}) D a(q) + \tau$$

$$a(q^-) = 0 \implies \dot{q}^+ = \Delta(q) \dot{q}^-$$

Cost function J encodes *task*:
– "leap onto box" or "leap over gap"

Minimize J by choosing control input τ:
– (biological or robotic) motor signals

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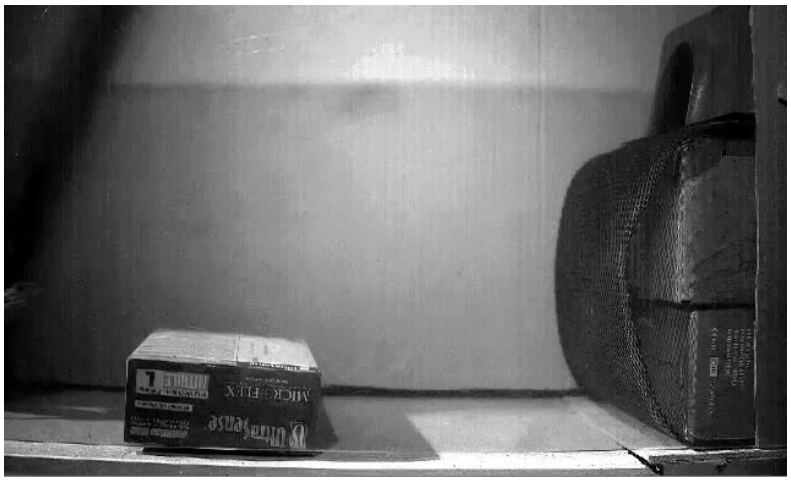

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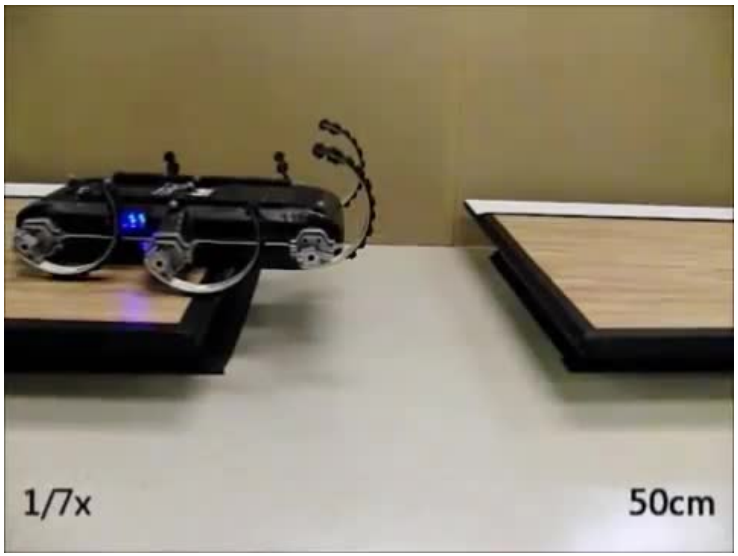

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Optimization with viscoelastic limbs

J is continuous and piecewise–smooth
– can apply scalable algorithms

Contribution from resolution of inconsistencies

Motivation: legged locomotion involves *intermittent* interaction
Predictions limited by pathologies in parsimonious models.

1. Mathematical "glue" removes discontinuities
   Yields reliable simulation algorithm and novel route to model reduction.

2. Restricting impact restitution resolves inconsistencies
   Yields scalable optimization algorithm and novel route to stabilization.

Future directions: predictions for robotics & biology
Effect of parameters and perturbations on gaits and maneuvers.

Predictions for robotics & biology

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Future directions: predictions for robotics & biology
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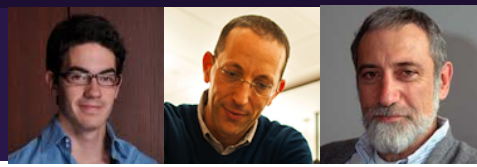
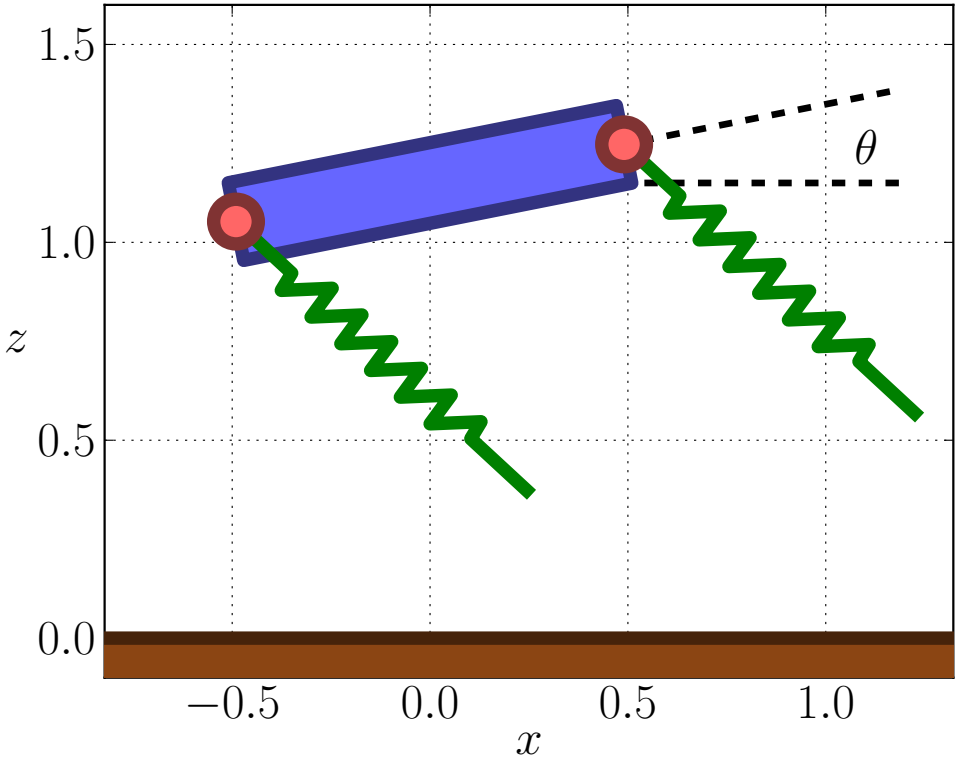
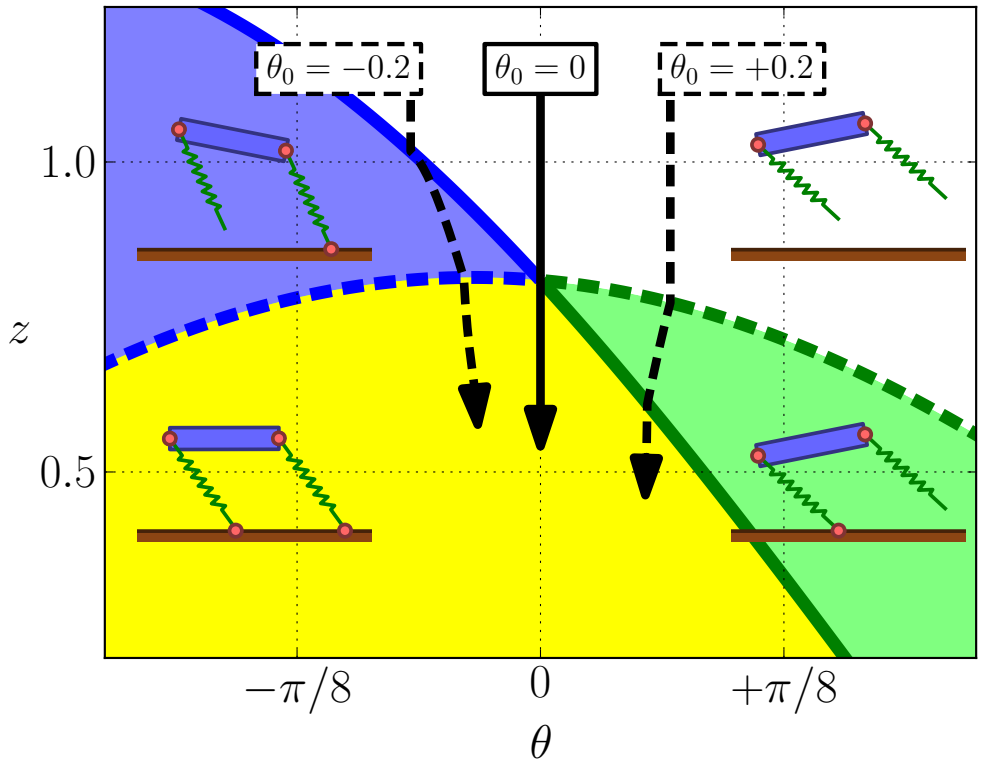
Implement stabilization mechanism







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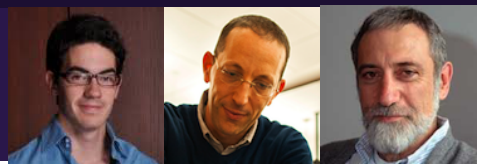
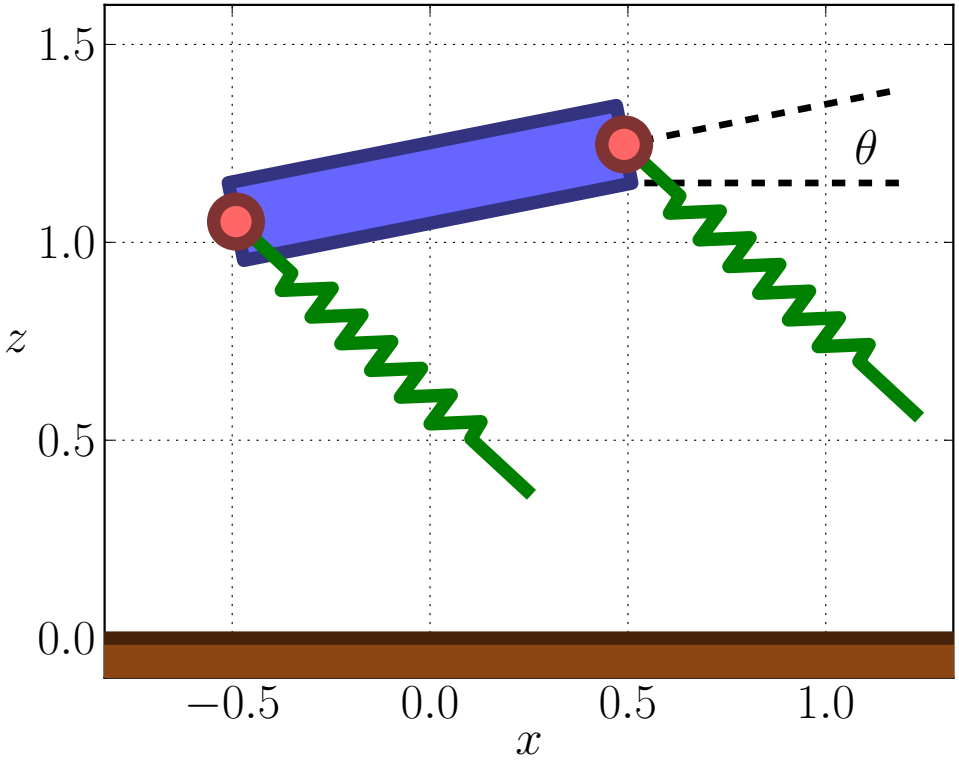
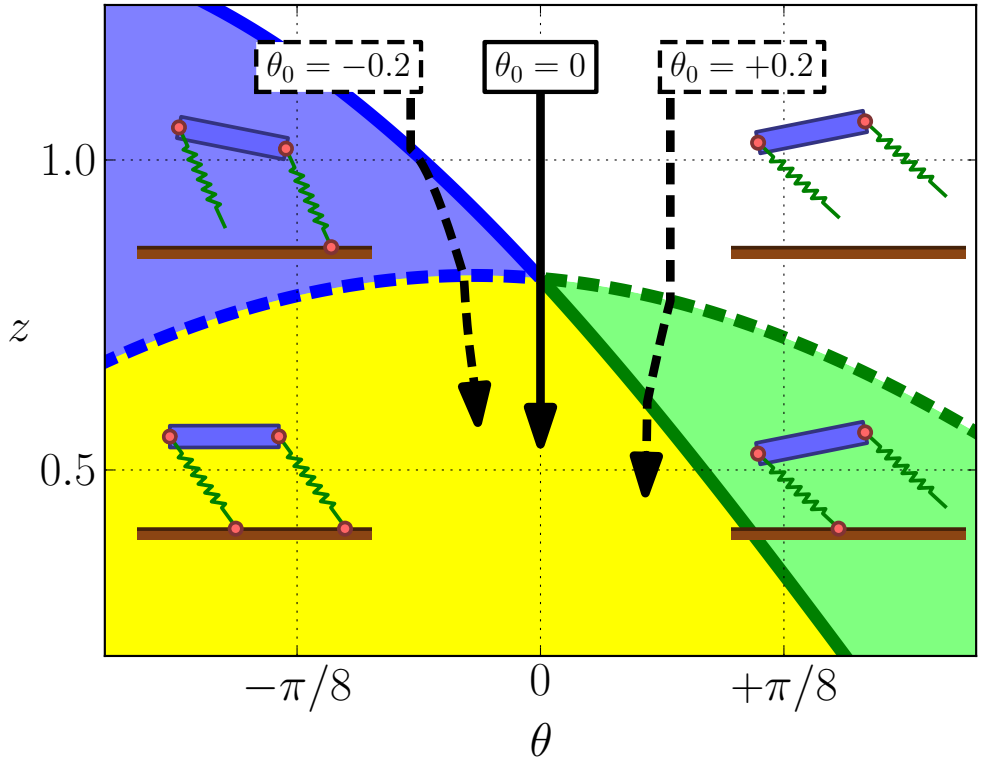


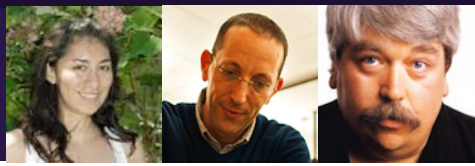




Predict passive stability

– rotations (pitch, roll) stabilize
– limb groupings synchronize

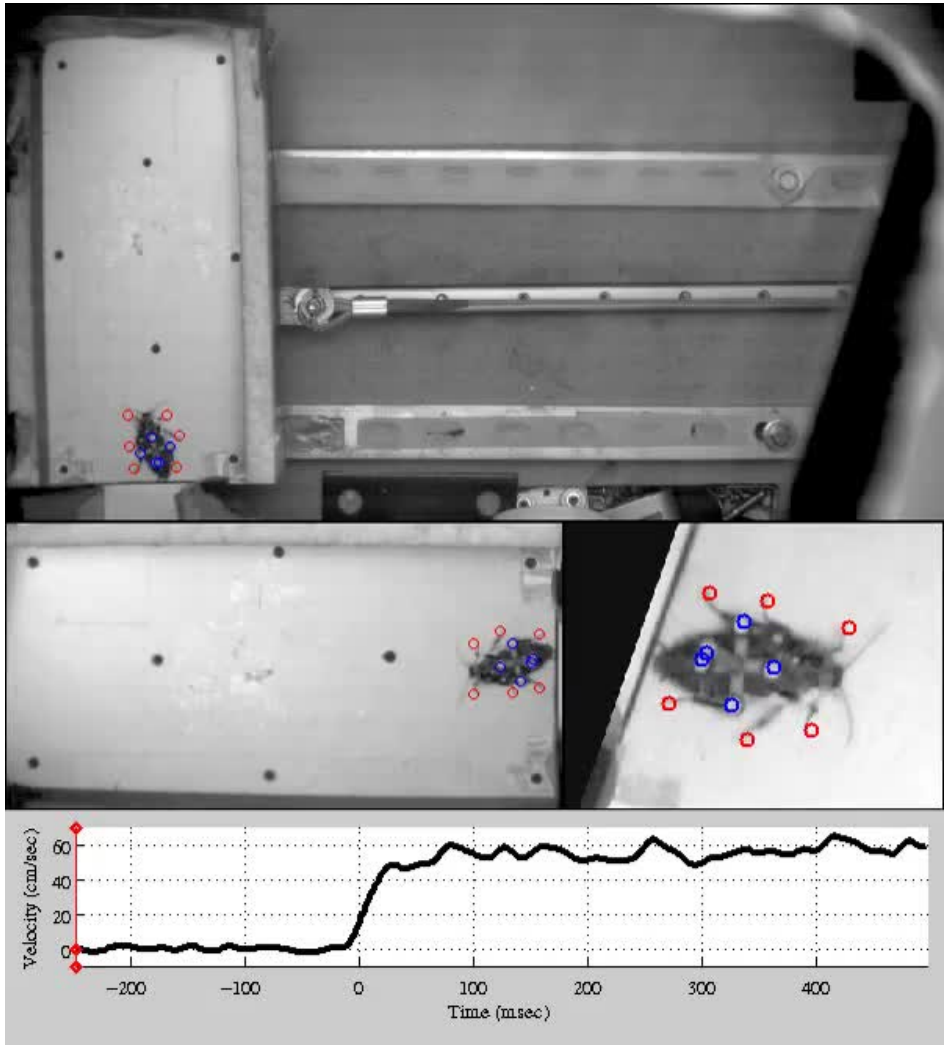
Identify perturbation recovery mechanism



observation

neural feedback appears at a delay

Revzen, Burden et al. BC 2013



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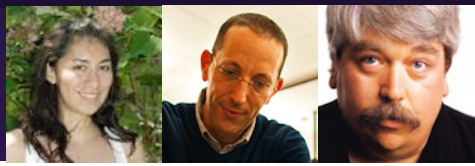

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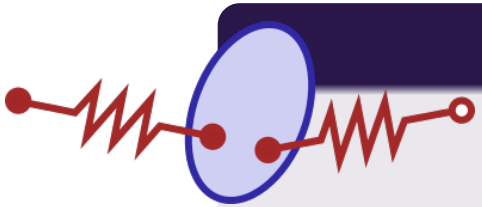
identification

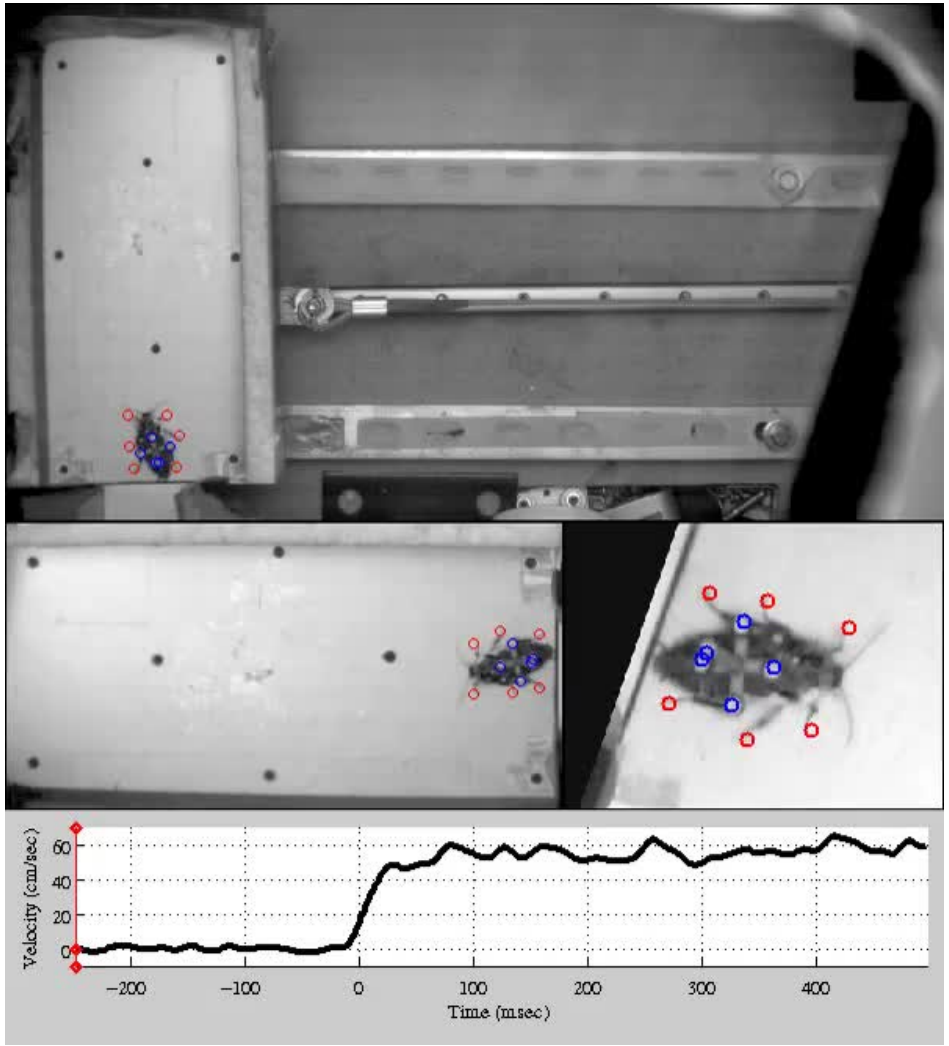


prediction

passive mechanics sensitive to inertia

Full et al. 2002

Burden, Revzen, Moore, Sastry, Full SICB 2013



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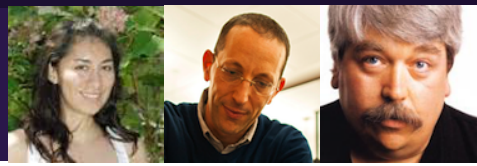

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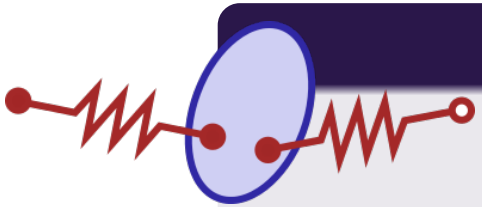

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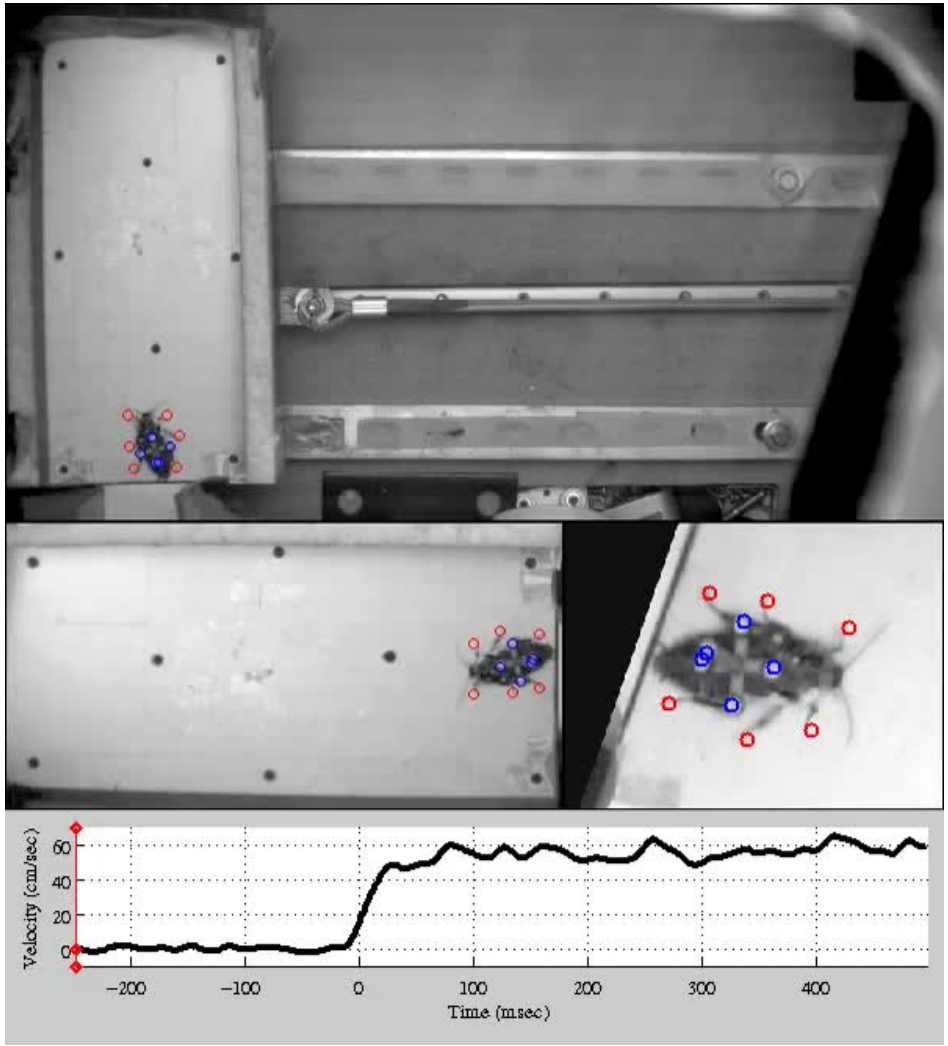
Check out Bora's poster!

#15 at 5pm

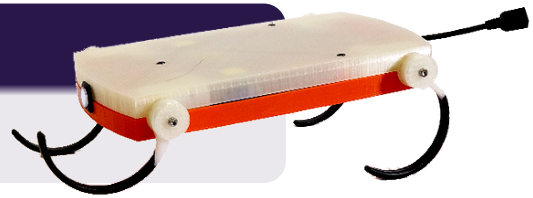
Burden, Revzen, Moore, Sastry, Full SICB 2013

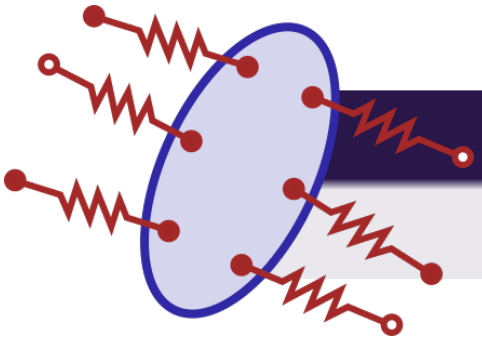


Challenge: integrating across hierarchies of models

physical system

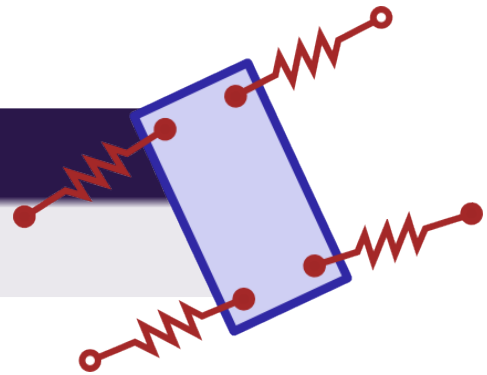
animal, robot

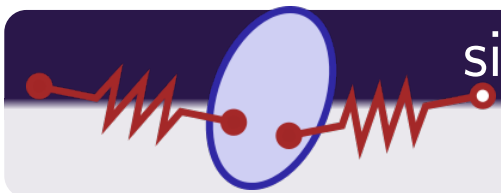




detailed model (*anchor*)

10–100 states, parameters



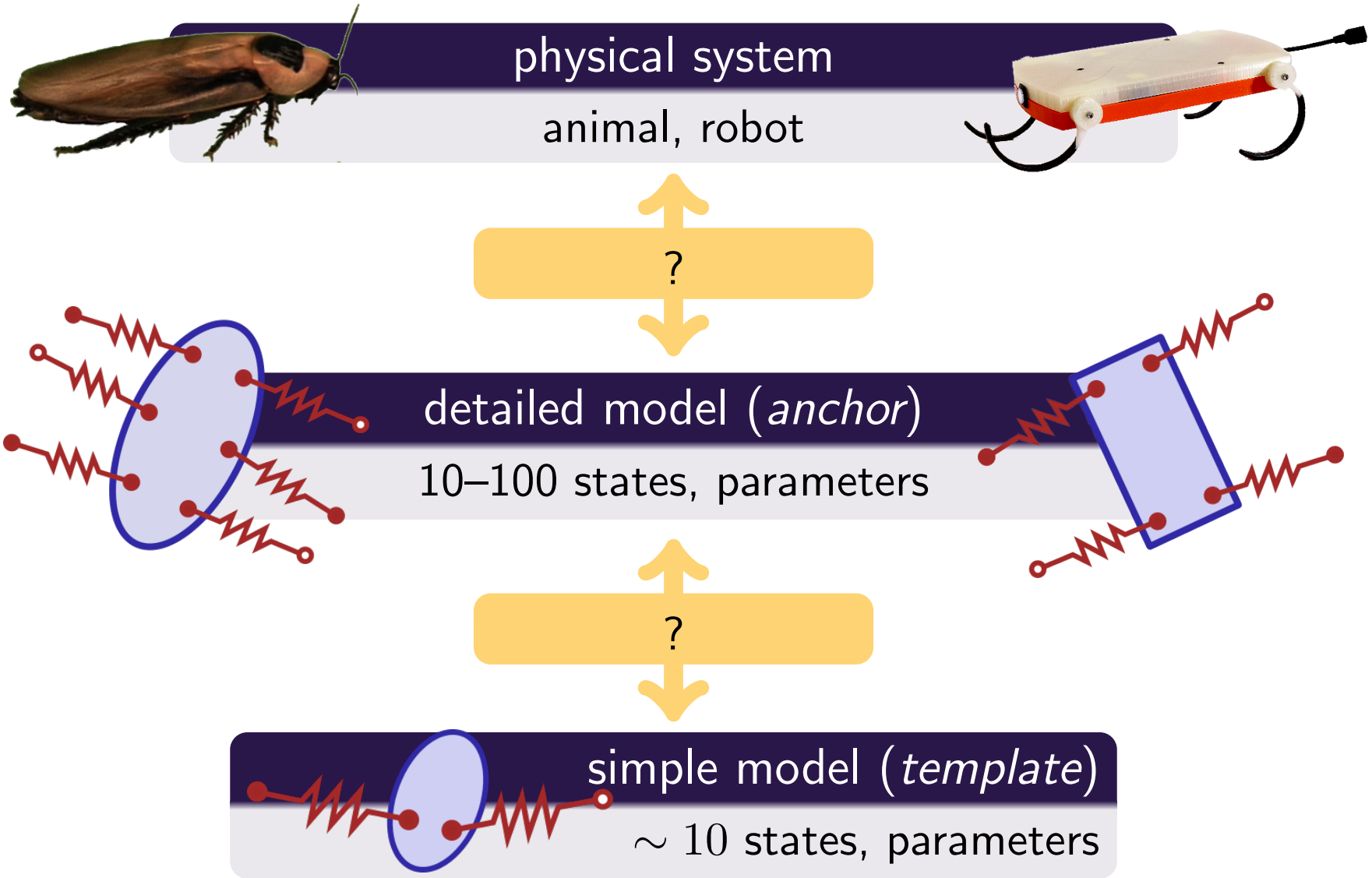


simple model (*template*)

~ 10 states, parameters

Full, Koditschek JEB 1999

Challenge: integrating across hierarchies of models



Full, Koditschek JEB 1999; Nishikawa *et al.* ICB 2007; Schwenk *et al.* ICB 2009

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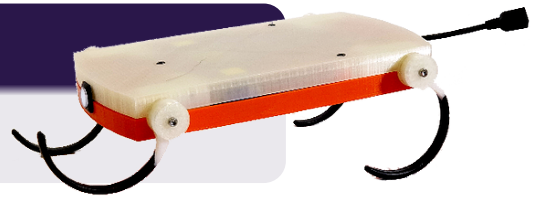
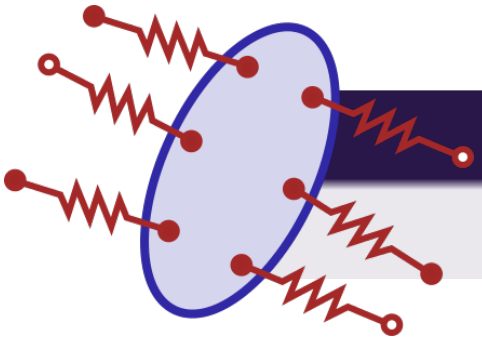


simulation, optimization



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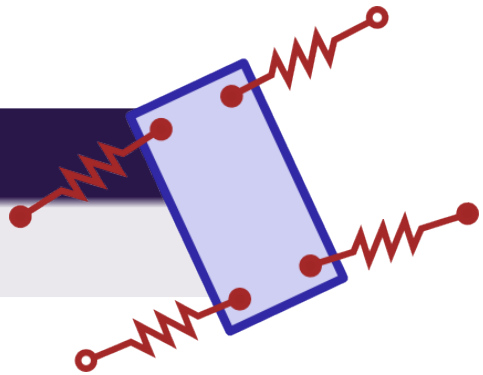
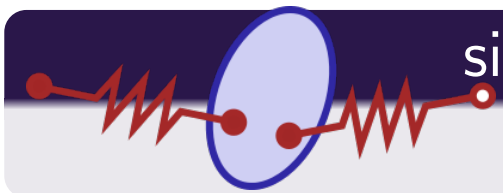


reduction, stabilization



simple model (*template*)

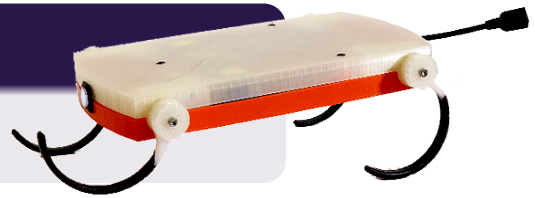
~ 10 states, parameters

Full, Koditschek JEB 1999; Nishikawa *et al.* ICB 2007; Schwenk *et al.* ICB 2009; Burden 2015

Opportunity: translate findings between disciplines

physical system

animal, robot

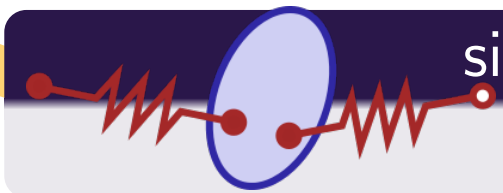


simulation, optimization

Parsimonious models enable direct translation

Indifferent to scale, material, or morphology

reduction, stabilization



simple model (*template*)

~ 10 states, parameters

Full, Koditschek JEB 1999; Nishikawa *et al.* ICB 2007; Schwenk *et al.* ICB 2009; Burden 2015

Discussion & Questions — Thanks for your time!

Parsimonious predictive models for legged locomotion

Resolving pathologies yields new algorithms and mechanisms.

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